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Methods of Finding All Factors of a Natural Number

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ABSTRACT

This study focused on "Methods of finding all factors of a natural number". The study was designed to show the simplest and precise methods of finding all factors of a natural number, and to point out the advantages of the simplest and precise methods of finding all factors of a natural number. The study found that the SPAM method and the method of using REM formulas are simple and precise methods to find all factors of a given natural number. By applying the preliminary concepts for the simplest and precise methods of finding all factors of a natural number, methods of finding all factors of a natural number, i.e. the SPAM method and method of using REM formulas, were developed in this study.

Keywords: Natural number, Prime factorization, REM, SPAM

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1. INTRODUCTION

When we deal with a set of natural numbers, the concept of factors of a certain natural number is remembered (Tomas, 2022). We frequently apply the concept of factors of a certain natural number for our day – to – day activities (MOE, 2010).

A natural number x is said to be a factor or divisor of a natural number y if and only if x divides y. It is simple to find all factors of a small natural number using thinking skill (Nagell, 2021). But it is difficult to find all factors of a large natural number using thinking skill (Tizazu, 2022). For instance; to list out all factors of 1,000,000, we need to test 1,000,000 is divisible or not divisible by a number from 1 to 1,000,000. This wastes our time and resources (Raji, 2019).

Since there is no simple and precise method to find all factors of a certain natural number, students and teachers add a number which is not a factor of that number, and miss a number that is a factor of that number during the time of finding all factors of a certain natural number (Rosen, 2020). To eliminate these two problems, this study was conducted.

This study is used to show the simple and precise methods of finding all factors of a given natural number, to point out the advantages of the simple and precise methods of finding all factors of a given natural number, as a reference for teachers/students when they teach/learn factors of a natural number.

This study is based on a core concept of number theory "a factor of any composite natural number is not out of the multiples of a prime factors of the number "(Sefinew, 2020). "No simple and premise method to find all factors of a given natural number" was taken as the gap of the previously conducted researches.

2. RESEARCH METHODS

This study used classroom observation and document analysis data gathering instruments to gather available data that were important to conduct the study. Class observations in classes 7-12 were used to understand the problem of this study. The following concepts were taken from different materials as the preliminary concepts of the study using document analysis.

Definition 2.1: A set of natural numbers, denoted by ℕ, is a set of numbers which is described by

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$\mathbb{N} = \{1, \ 2, \ 3, \ 4, \ldots\}$ (Cameron, 2019).

Definition 2.2: Let x and y be two natural numbers. Then x is said to be a factor of y if and only if there exists a natural number t such that $y = tx$. This means that x is a factor of y if and only if x divides y (Fine, et al., 2017).

Example 2.1: 7 is a factor of 105 since there exists a natural number 15 such that 105 = 15 × 7.

Definition 2.3: A natural number is said to be

a. Prime number if it has exactly two distinct factors, namely 1 and itself.
b. Composite number if it has more than two distinct factors (Bordelles, 2022).

Remark 2.1: 1 is neither a prime nor a composite number (Zhao, 2017).

Example 2.2

1. Consider a natural number 13. Since 13 is divisible only by 1 and itself, it is a prime number.
2. Consider the natural number 16. The factors of 16 are 1, 2, 4, 8 and 16. Since 16 has five distinct factors, it is a composite number.

Definition 2.4: When a number is expressed as a product of its prime factors, the expression is called the prime factorization of a number (Fine, et al., 2017).

There are two methods to find the prime factorization of a number. These are

a. Tree Diagram Method
b. Tabular method (William, 2020).

Let us see these methods in the following examples.

Example 2.3

1. From Table 2.4.1, the prime factorization of 4500 is $2^2 \times 3^2 \times 5^3$.
2. a. From Figure 2.4.1, the prime factorization of 64 is 2^6.
   b. From Figure 2.4.2, the prime factorization of 1000 is $2^3 \times 5^3$.

Table 1

2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

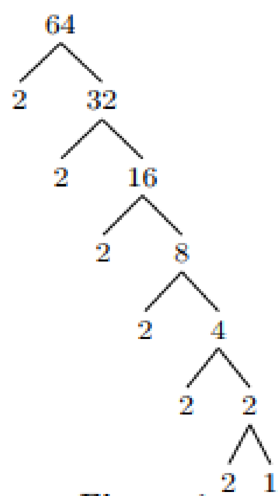

Figure 1

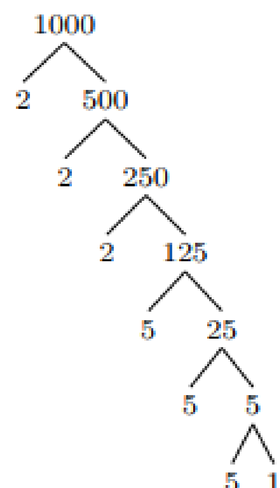

Figure 2

Definition 2.5: If a prime factorization of a natural number m is $p_1^{a_1} x \, p_2^{a_2} x \cdots x \, p_n^{a_n}$, then the total **number of factors** of m, denoted by T(m), is calculated by

$$T(m) = (a_1 + 1)x(a_2 + 1)x \cdots x(a_n + 1)$$ [3] [13].

Example 2.4 From Example 2.3, the prime factorization of

- 4500 is $2^2 \times 3^2 \times 5^3$
- 64 is 2^6
- 1000 is $2^3 \times 5^3$.

Then the

1. $T(4500) = (2 + 1)(2 + 1)(3 + 1) = 36$

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2. T (64) = (6 + 1) = 7
3. $T(1000) = (3+1)(3+1) = 16$

Definition 2.6: Let F and G be the finite sets of natural numbers. Then the **SPAM** of F and G, denoted by F $\boxed{s}$ G, is given by

$F \boxed{s} G = \{y: y = a \times b \text{ for all } a \in F \text{ \& all } b \in G, \text{and } \times \text{ is an ordinary multiplication}\}$.

Example 2.5: Let $F = \{1, 2, 5\}$ and $G = \{3, 4\}$, then the SPAM of F and G is given by

$F \boxed{s} G = \{1x3, 1x4, 2x3, 2x4, 5x3, 5x4\}$

$\Longrightarrow F \boxed{s} G = \{3, 4, 6, 8, 15, 20\}$

Remark 2.2: Let F, G and H be the finite sets of natural numbers, then

- F $\boxed{s}$ G = G $\boxed{s}$ F. This is known as the commutative property.
- [F $\boxed{s}$ G] $\boxed{s}$ H = F $\boxed{s}$ [G $\boxed{s}$ H]. This is known as the associative property.
- The number of elements of F $\boxed{s}$ G is less than or equal to the product of the number of elements of F & the number of elements of G.

Theorem 2.1. If p is a prime factor of a natural number N, then the multiples of $P \leq N$ can be the factors of N. This idea is very important to develop the third steps in REM and SPAM methods (Tadesse, 2021).

3. RESULTS AND DISCUSSION

This study developed two methods which are important to find all factors of a given natural number using the stated preliminary concepts. These methods are

a. SPAM method and
b. Method of using REM formulas.

These methods minimize anxiety and fatigue when students and teachers find all factors of a given natural number.

A. SPAM method

To find all factors of a given natural number using the SPAM method, we must apply the following steps.

Step 1: Express the given natural number in prime factorization.
Step 2: Find the total number of factors of a given natural number.
Step 3: List all factors of a given natural number using the concept of SPAM. In this step, we have the following three cases.

Case – one

Suppose that the prime factorization of a natural number m is $p_1^{a_1}$. Then the set of all factors of m is given by

$$B(m) = F \boxed{s} \{1\}$$

where $F = \{p_1^{i}: i = 0, 1, 2, \cdots, a_1\}$.

Case – two

Suppose the prime factorization of a natural number n is $p_1^{a_1} x\, p_2^{a_2}$. Then the set of all factors of n is given by

$$B(n) = F \boxed{s} G$$

where $F = \{p_1^{i}: i = 0, 1, 2, \cdots, a_1\}$ and
$G = \{p_2^{j}: j = 0, 1, 2, \cdots, a_2\}$.

Case – three

Suppose the prime factorization of a natural number w is $p_1^{a_1} x\, p_2^{a_2} x\, p_3^{a_3}$. Then the set of all factors of w is given by

$$B(w) = [F \boxed{s} G] \boxed{s} H$$

where $F = \{p_1^{i}: i = 0, 1, 2, \cdots, a_1\}$,
$G = \{p_2^{j}: j = 0, 1, 2, \cdots, a_2\}$ and
$H = \{p_3^{k}: k = 0, 1, 2, \cdots, a_3\}$

Remark 3.1.1

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- $B(m), B(n)$ and $B(w)$ denote the set of all factors of natural numbers m, n and w respectively.
- In general, if the prime factorization of a natural number z is ${p_1}^{a_1} x\, {p_2}^{a_2} x \cdots x\, {p_n}^{a_n}$, then the set of all factors of z is given by

$$B(z) = F_1 \boxed{s} F_2 \boxed{s} \cdots \boxed{s} F_n$$

where $F_k = \{{p_k}^i : i = 0, 1, \cdots, a_k\}$ and $k = 1, 2, \cdots, n$.

Example 3.1.1: Using SPAM method, let us find all factors of 64, 1000 and 4500.

a. From examples 2.3 and 2.4,
   i. The prime factorization of 64 is 2^6.
   ii. The total number of factors of 64 is 7.

Let F = $\{2^i : i = 0, 1, 2, 3, 4, 5. 6\}$
$\Longrightarrow F = \{1, 2, 4, 8, 16, 32, 64\}$
Using case one (i.e. multiplying every element of F by 1),
$B(64) = F \boxed{s} \{1\} = \{1, 2, 4, 8, 16, 32, 64\}$
Hence, all factors of 64 are 1, 2, 4, 8, 16, 32 and 64.

b. From examples 2.3 and 2.4,
   i. The prime factorization of 1000 is $2^3 \times 5^3$.
   ii. The total number of factors of 1000 is 16.

Let $F = \{2^i : i = 0, 1, 2, 3\} = \{1, 2, 4, 8\}$
$G = \{5^j : j = 0, 1, 2, 3\} = \{1, 5, 25, 125\}$
Using case two (i.e. multiplying every element of F by every element of G),
$B(1000) = F \boxed{s} G = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500, 1000\}$
Hence, all factors of 1000 are 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, 500 and 1000.

c. From examples 2.3 and 2.4,
   i. The prime factorization of 4500 is $2^2 \times 3^2 \times 5^3$.
   ii. The total number of factors of 4500 is 36.

Let $F = \{2^i : i = 0, 1, 2\} = \{1, 2, 4\}$
$G = \{3^j : j = 0, 1, 2\} = \{1, 3, 9\}$
$H = \{5^k : k = 0, 1, 2, 3\} = \{1, 5, 25, 125\}$
Using case two (i.e. multiplying every element of F by every element of G) and case three (i.e. multiplying every element of F $\boxed{s}$ G by every element of H),
$B(4500) = [F \boxed{s} G] \boxed{s} H$
$\Longrightarrow B(4500) = \{1, 2, 3, 4, 6, 9, 12, 18, 36\} \boxed{s} H$
$\Longrightarrow B(4500) = \{$1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 30, 36, 45, 50, 60, 75, 90, 100, 125, 150, 180, 225, 250, 300, 375, 450, 500, 750, 900, 1125, 1500, 2250, $4500\}$
Hence, all factors of 4500 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 30, 36, 45, 50, 60, 75, 90, 100, 125, 150, 180, 225, 250, 300, 375, 450, 500, 750, 900, 1125, 1500, 2250 and 4500.

B. Method of using REM formulas

To find all factors of a given natural number using REM formulas, we must apply the following steps.
Step 1: Express the given natural number in prime factorization.
Step 2: Find the total number of factors of a given natural number.
Step 3: List all factors of a given natural number using REM formulas. In this step, we have the following three formulas.

R – formula
Let the prime factorization of a natural number x is ${p_1}^{a_1}$, then we can find all factors of x by using the following formula.

$$R_i = {p_1}^{i-1}$$

where $i = 1, 2, 3, \cdots, (a_1 + 1)$. This formula is said to be the **R – formula.**

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E – formula
Let the prime factorization of a natural number y is $p_1^{a_1} x\, p_2^{a_2}$, then we can find all factors of y by using the following formula.

$$E_j = R_i \text{ x } p_2^{j-1}$$

where R_i are the factors of $p_1^{a_1}$; which are calculated using R – formula & $j = 1, 2, 3, \cdots, (a_2 + 1)$. This formula is said to be **E – formula.**

M – formula
Let the prime factorization of a natural number z is $p_1^{a_1} x\, p_2^{a_2} x\, p_3^{a_3}$, then we can find all factors of z by using the following formula.

$$M_k = E_j \text{ x } p_3^{k-1}$$

where E_j are the factors of $p_1^{a_1} x\, p_2^{a_2}$; which are calculated using E – formula & $k = 1, 2, 3, \cdots,$ $(a_3 + 1)$. This formula is said to be the **M – formula.**

Remark 3.2.1

1. The three formulas are said to be REM formulas.
2. We apply REM formulas when a natural number has at most three distinct prime factors. But, by applying the pattern of REM formulas repeatedly, we can find all factors of a given natural number that has more than three distinct prime factors.

Example 3.2.1: Using REM formulas, let us find all factors of natural numbers 32, 36 and 84.

a. Step 1: The prime factorization of 32 is 2^5.
Step – 2: $T(32) = 6$.
Step -3: Let us find all factors of 32 using the R – formula.
$R_i = 2^{i-1}$; where $i = 1, 2, 3, \cdots, 6$
$\Rightarrow R_1 = 2^{1-1} = 1$
$R_2 = 2^{2-1} = 2$
$R_3 = 2^{3-1} = 4$
$R_4 = 2^{4-1} = 8$
$R_5 = 2^{5-1} = 16$
$R_6 = 2^{6-1} = 32$
Hence, all factors of 32 are 1, 2, 4, 8, 16 and 32.

b. Step – 1: The prime factorization of 36 is $2^2 x 3^2$.
Step – 2: $T(36) = 9$.
Step -3: Let us find all factors of 36 using the E – formula.
Firstly, let us find the factors of 2^2 using the R – formula.
$R_i = 2^{i-1}$; where $i = 1, 2, 3$
$\Rightarrow R_1 = 1$, $R_2 = 2$ and $R_3 = 4$
Secondly, let us find the factors of 2^2 x 3^2 using the E – formula.
$E_j = R_i \text{ x } 3^{j-1}$; where $j = 1, 2, 3$

- $E_1 = R_i \text{ x } 3^{1-1} = R_i$

Since the values of i are 1, 2 and 3, then

$$E_1 = \begin{cases} R_1 = 1 \text{ if } i = 1 \\ R_2 = 2 \text{ if } i = 2 \\ R_3 = 4 \text{ if } i = 3 \end{cases}$$

$$\Rightarrow E_1 = 1, 2 \text{ and } 4$$

- $E_2 = R_i \text{ x } 3^{2-1} = 3R_i$

Since the values of i are 1, 2 and 3, then

$$E_2 = \begin{cases} 3R_1 = 3x1 = 3 \text{ if } i = 1 \\ 3R_2 = 3x2 = 6 \text{ if } i = 2 \\ 3R_3 = 3x4 = 12 \text{ if } i = 3 \end{cases}$$

$\Rightarrow E_2 = 3, 6$ and 12

- $E_3 = R_i \text{ x } 3^{3-1} = 9R_i$

Since the values of i are 1, 2 and 3, then

$$E_3 = \begin{cases} 9R_1 = 9x1 = 9 \text{ if } i = 1 \\ 9R_2 = 9x2 = 18 \text{ if } i = 2 \\ 9R_3 = 9x4 = 36 \text{ if } i = 3 \end{cases}$$

$\Rightarrow E_3 = 9, 18$ and 36

Hence, the values of E_1, E_2 and E_3 are all factors of 36. That means, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

c. **Step – 1:** The prime factorization of 84 is 2^2x3x7.

Step – 2: $\tau(84) = 12$.

Step -3: Let us find all factors of 84 using the M – formula.

Firstly, let us find the factors of 2^2 using the R – formula.

$R_i = 2^{i-1}$; where $i = 1, 2, 3$

$\Rightarrow R_1 = 1$, $R_2 = 2$ and $R_3 = 4$

Secondly, let us find the factors of 2^2x3 using E – formula.

$E_j = R_i \text{ x } 3^{j-1}$; where $j = 1, 2$

- $E_1 = R_i \text{ x } 3^{1-1} = R_i$

Since the values of i are 1, 2 and 3, then

$$E_1 = \begin{cases} R_1 = 1 \text{ if } i = 1 \\ R_2 = 2 \text{ if } i = 2 \\ R_3 = 4 \text{ if } i = 3 \end{cases}$$

$\Rightarrow E_1 = 1, 2$ and 4

- $E_2 = R_i \text{ x } 3^{2-1} = 3R_i$

Since the values of i are 1, 2 and 3, then

$$E_2 = \begin{cases} 3R_1 = 3x1 = 3 \text{ if } i = 1 \\ 3R_2 = 3x2 = 6 \text{ if } i = 2 \\ 3R_3 = 3x4 = 12 \text{ if } i = 3 \end{cases}$$

$\Rightarrow E_2 = 3, 6$ and 12

Thirdly, let us apply the M – formula to find the factors of $2^2 \text{ x } 3 \text{ x } 7$.

$M_k = E_j \text{ x } 7^{k-1}$; where $k = 1, 2$

- $M_1 = E_j \text{ x } 7^{1-1} = E_j$

Since the values of j are 1 and 2, then

$$M_1 = \begin{cases} E_1 = 1, 2 \text{ and } 4 \text{ if } j = 1 \\ E_2 = 3, 6 \text{ and } 12 \text{ if } j = 2 \end{cases}$$

$\Rightarrow M_1 = 1, 2, 3, 4, 6$ and 12

- $M_2 = E_j \text{ x } 7^{2-1} = 7E_j$

Since the values of j are 1 and 2, then

$$M_2 = \begin{cases} 7E_1 = 7, 14, 28 \text{ if } j = 1 \\ 7E_2 = 21, 42, 84 \text{ if } j = 2 \end{cases}$$

$\Rightarrow M_2 = 7, 14, 21, 28, 42$ and 84

Hence, the values of M_1 and M_2 are all factors of 84. That means that the factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84.

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4. CONCLUSION

This study focused on methods of finding all factors of a given natural number. These methods are the SPAM method and method of using REM formulas. The main advantage of these methods is minimizing anxiety and fatigue when students and teachers list out all factors of a given natural number.

Before we deal with methods of finding all factors of a natural number, we must have the concepts of a set of natural numbers, factor or divisor of a natural number, prime and composite numbers, prime factorization of a natural number, formula for finding total number of factors of a natural number, and SPAM of two finite sets of natural number.

Abbreviations

SPAM denotes 'Sertsepetros Aschale Moges', who is a child of this researcher. He was born on November 24, 2007 E.C.

REM denotes 'Remembering Etenesh Moges (1982 – 2000 E.C.)", who is a sister of this researcher. She died in November 24/2000 E.C. She studied her education until grade seven with good rank. Sertsepetros Aschale and Etenesh Moges have great historical relationships with this researcher. This work was done to memorize the historical relation of Sertsepetros Aschale and Etenesh Moges.

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REFERENCES

[1] Bordelles, O. (2022, June 24). On certain sums of number theory. International Journal of Number Theory, 18(09), 2053–2074. https://doi.org/10.1142/s1793042122501056.
[2] Cameron, P. J. (2019). A course on number theory. English: Cambridge University press.
[3] Cleveland, T. (2018, April 11). Number Theory. Scientific e-Resources. http://books.google.ie/books?id=o-PEDwAAQBAJ&printsec=frontcover&dq=number+theory&hl=&cd=6&source=gbs_api.
[4] Fine, B., Gaglione, A., Moldenhauer, A., Rosenberger, G., & Spellman, D. (2017, September 11). Algebra and Number Theory. Walter de Gruyter GmbH & Co KG. http://books.google.ie/books?id=7-Y3DwAAQBAJ&printsec=frontcover&dq=number+theory&hl=&cd=3&source=gbs_api
[5] MOE (2010). Mathematics book for grade 9-12. Ethiopia: Aster nega publishing enterprise.
[6] Nagell, T. (2021, July 21). Introduction to Number Theory. American Mathematical Soc. http://books.google.ie/books?id=Znc5EAAAQBAJ&printsec=frontcover&dq=number+theory&hl=&cd=4 &source=gbs_api.
[7] Raji, W. (2019). An introduction course in elementary number theory. English: Oxford University press.
[8] Rosen, K. H. (2020). Elementary number theory and its applications. English: Addison Wesley publishing company.
[9] Sefinew, A. (2020). Teaching Mathematics. Ethiopia: Gondar press.
[10] Tadesse, Y. (2021). Teaching methodology for high school Mathematics. Ethiopia: Bahir Dar press.
[11] Tizazu, W. (2022). Teaching methodology for elementary school Mathematics. Ethiopia: Debre Markos press.
[12] Tomas, Y. (2019). Mathematics education for lower grade level. Ethiopia: Debre Birhan press.
[13] Vitmak, Y. (2017). Elementary concept of number theory. South Africa: Pretoria press.
[14] William, Z. (2020). A first course in Number theory. USA: New York press.
[15] Zhao, L. (2017, September). On ternary problems in additive prime number theory. Journal of Number Theory, 178, 179–189. https://doi.org/10.1016/j.jnt.2017.02.008.