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Application of *Runge Kutta Fehlberg* (RKF45) Method as a Numerical Analysis to SIR Model of Tuberculosis Transmission in Central Java

Nur Alisa[1], Ade Ima Afifa Himayati[2], Findasari[3]
[1,2,3] Department of Mathematics, Faculty of Science and Technology, Universitas Muhammadiyah Kudus, Kudus Indonesia
[1]32021130002@std.umku.ac.id, [2]adeimaafifa@umkudus.ac.id, [3]findasari@umkudus.ac.id

ABSTRACT

This study uses a three-compartment SIR model for the spread of tuberculosis in Central Java. Changes in individuals detected, recovered, died naturally, and died due to tuberculosis affect disease transmission. Numerical simulations were used to validate the analysis results and identify the main parameters that most contribute to disease spread among susceptible, infected, quarantined, and recovered individuals. The numerical method used is *Runge Kutta Fehlberg*. Using this method, a quantitative description of the number of susceptible, infected, and recovered populations is obtained, which can assist the Central Java Health Office in its efforts to prevent and control the spread of tuberculosis. The SIR model obtained from determining the parameters is then solved using the Runge Kutta Fehlberg method. The results obtained using data from 2021-2023 show that the initial value for *Susceptible* is 111.120.397, the initial value *for Infected* is 157.024, and the initial value *for Recovered* is 38.452, with a birth rate parameter of 0,013043, a natural death rate of 0,001287, a tuberculosis death rate of 0,041376, a transmission rate from susceptible to infected of 0,001411, and a recovery rate from infected to recovered of 0,24488 people. In year 50, there were 35.073.325 *susceptible* individuals, the number *of infected individuals* in the 50th year is 0,04, and the number of *recovered individuals* in the 50th year is 74.774. The number of tuberculosis infection cases decreases from year to year.

Keywords: Epidemiology, Mathematical Modeling, Numerical Analysis, Runge Kutta Fehlberg, SIR, Tuberculosis.

Corresponding Author:
Nur Alisa
Department of Mathematics,
Universitas Muhammadiyah Kudus,
Jl. Ganesha Raya No.I, Purwosari, Kec. Kota Kudus, Kabupaten Kudus, Jawa Tengah 59316.
32021130002@std.umku.ac.id

1. INTRODUCTION

Along with the development of science and technology, the discipline of mathematics has developed rapidly. Mathematics can be applied in various sectors. For example, biomedical mathematics is the application of mathematics in biology (Hurit et al, 2022). In mathematical modeling, mathematical models can describe complex abstract events and become *problem solving* in the world of health. Mathematical models are very diverse, and as mathematics develops, many mathematical models are created(Ndii, 2018). A widely used mathematical model that produces accurate solutions is the *Susceptible, Infected, Recovered* (SIR) mathematical model. This mathematical model is used to predict a phenomenon based on transmission (Moujahid & Vadillo, 2021). The SIR model is commonly used to predict the transmission of infectious diseases within a certain period of time, such as in the research by (Sifriyani & Rosadi, 2020) to see the estimated reproduction number of Covid-19. The SIR mathematical model is one of the models used in epidemiology to represent the spread of infectious diseases in a particular population by describing the dynamics of the number of individuals in the population with three components, namely S (*Susceptible)* individuals who are vulnerable to infection, I (*Infected)* individuals who are infected with the disease, and R (*Recovered)* individuals who have recovered from the disease. Epidemiological models are very suitable for presenting the transmission rate of infectious diseases such as tuberculosis (TB) because the SIR model can present information that is in line with the data obtained, namely that there are three components: *Susceptible, Infected*, and *Recovered* (Yudasubrata, 2018).

In developing countries such as Indonesia, dealing with infectious diseases is a daunting task. According to the Indonesian Central Statistics Agency (BPS), Indonesia's population in 2021-2023 will be 281,603,800. Tuberculosis in Indonesia is a major concern. Tuberculosis (TB) is an infectious disease caused by the *Mycobacterium tuberculosis* bacterium, which is transmitted through the air and attacks organs such as the lungs, kidneys, brain, spine, and lymph nodes. According to the WHO, throughout 2022, the second deadliest infectious disease after Covid-19 was tuberculosis. According to TB Indonesia (TB Indonesia, 2024), Indonesia ranks second with the highest number of TB cases in the world after India, followed by China. With the highest number of TB cases nationally, Central Java province has a fairly high number of TB cases. The high prevalence of TB cases certainly requires special attention and effective

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measures to prevent the spread of TB. In dealing with the spread of TB, it can be done not only in the health sector, but also by utilizing mathematical modeling to help predict TB transmission using the SIR mathematical model(Nafsi & Rahayu, 2020). The SIR model was first introduced by Kermack and McKendrick in 1927 and is still widely used in epidemiological studies today(Ningsih & Mungkasi, 2020). In this model, the rate of movement of individuals between subpopulations is described using a system *of ordinary differential equations* (ODE)(Purnomo Dwi, 2012). By using the SIR model systematically, we can observe the dynamics of TB spread.

Research related to mathematical modeling, namely the SIR model, includes Bahari et al.'s (Bahari et al, 2021) study on the SIR spread of tuberculosis in Central Java. followed by numerical research by Rif'at et al. (Rif'at et al, 2022) on the analysis of the spread rate of Covid-19 using the SIR epidemiological mathematical model and fourth-order *Runge Kutta* in the city of Surabaya. Furthermore, there is numerical solution research by Anwar et al. (Anwar et al, 2023) on the numerical solution of the SIR model in the spread of tuberculosis using the *Runge Kutta Fehlberg* (RKF45) method.

The *Runge Kutta Fehlberg* (RKF45) method is one of the numerical methods widely used to solve differential equations (Al-Bugami & Al-Juaid, 2020). RKF45 is a numerical algorithm resulting from the modification of the fourth-order *Runge Kutta* method and the fifth- order *Runge Kutta* method used in solving ordinary differential equations with a high degree of accuracy (Ghazal & Hussain, 2021). High accuracy occurs because this model has six constants. The RKF45 method is a frequently used and popular numerical method (Anwar et al, 2023). According to the introduction, this study aims to determine the SIR model and the numerical solution of the SIR mathematical model in the transmission of tuberculosis in Central Java by applying the RKF45 numerical method(Suryaningrat et al, 2020). The solution obtained will be considered by the Central Java Provincial Health Office in responding to tuberculosis cases by eradicating transmission to minimize the spread of tuberculosis in Central Java Province.

2. RESEARCH METHOD

This research was conducted to see the success of using the RKF45 mixed analysis method to solve the SIR Mathematical model on the spread of tuberculosis disease over a period of 3 years. This research was completed through five stages, namely planning, implementation, data collection, modeling, and application of the RKF45 method. This research was conducted in Central Java using tuberculosis patient data from 2021 to 2023.

The materials used for this study are secondary data from the Central Statistics Agency (BPS) of Central Java Province and the Central Java Provincial Health Office which were obtained in 2024. The collected quantitative data was processed through the SIR mathematical model analysis stage to obtain an overview of the spread of the disease from 2021 to 2023. The next stage is to enter the model into the RKF45 equation using Matlab software, this stage is the final stage in the analysis which produces predicted results for the number of susceptible, infected, and recovered individuals in the next 50 years.

3. RESULTS AND DISCUSSION

The mathematical model for the spread of tuberculosis (SIR) in Central Java can be observed through secondary data calculations adapted to the formulas used in the SIR model analysis. This model is expected to provide an overview and prediction of the spread of tuberculosis in Central Java, enabling responsible parties to provide prevention and control measures for tuberculosis cases in the region. The SIR model analysis for tuberculosis in Central Java is presented in the table below:

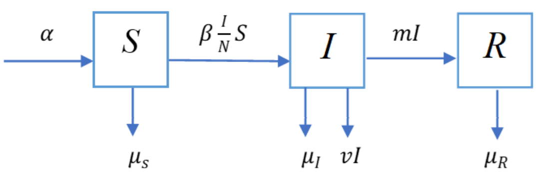

Figure 1. SIR Tuberculosis Model

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The image above is a picture of the tuberculosis SIR model that corresponds to the conditions of tuberculosis spread in Central Java from 2021 to 2023. Explanations of the mathematical symbols can be seen in tables 1 and 2 below. From the SIR tuberculosis mathematical model image above, the following model equation is obtained:

$$\frac{dS}{dt} = \alpha - \beta\frac{I}{N}S - \mu_s$$

$$\frac{dI}{dt} = \beta\frac{I}{N}S - mI - \mu_I - vI$$

$$\frac{dR}{dt} = mI - \mu_R$$

Table 1. Variable Of SIR Model

Variables	Definition	Value	Information
S_0	Early susceptible	111.120.397	Data on the number of individuals susceptible to tuberculosis in Central Java in 2021-2023
I_0	Early infected	157.024	Data on the number of individuals infected (positive) with tuberculosis in Central Java in 2021-2023
R_0	Earl recovered	38.452	Data on the number of individuals recovered from tuberculosis in Central Java in 2021-2023

Source: Data processed by researchers

Table 1 lists the variable names used in SIR modeling. The obtained data are then labeled with variables, such as S for the number of individuals susceptible to the disease, I for individuals infected with the disease, and R for individuals recovered. The small number 0 below the symbol represents the initial data or data before analysis

Table 2. Parameters Of SIR Model

Parameters	Definition	Formulation	Value
α	Birth rate	$\alpha = \frac{total\ of\ birth}{total\ population}$	0,013043
μ	Natural death rate	$\mu = \frac{natural\ death\ total}{total\ population}$	0,001287
v	Death rate from tuberculosis	$v = \frac{total\ of\ deaths\ due\ to\ TB}{total\ TB\ infections}$	0,041376
β	Transmission rate from susceptible to infected	$\beta = \frac{total\ of\ infections}{total\ of\ population}$	0,001411
m	rate of recovery from infected to cured	$m = \frac{total\ of\ recovered}{total\ of\ infections}$	0,24488

Source: Data processed by researchers

Table 2 lists the parameter values required for the SIR and RKF45 model analyses. The symbols used have their own meanings; the SIR and RKF45 model analyses require five parameters that represent characteristics within the population.

From the results of the parameter calculations in the table 2. Substitute into the Central Java tuberculosis SIR model, with the results in the equation below:

$$\frac{dS}{dt} = 0{,}01304 - 0{,}00141\frac{I}{N}S - 0{,}001284S$$

$$\frac{dI}{dt} = 0{,}00141\frac{I}{N}S - 0{,}24488I - 0{,}00128I - 0{,}04137I$$

$$\frac{dR}{dt} = 0{,}24488I - 0{,}00128R$$

The SIR mathematical model equation above will be used for the RKF45 numerical analysis method. The RKF45 numerical analysis equation can be seen in the equation below:

$$\hat{S}_{t+1} = S_i + \frac{16}{135}k\hat{S}_1 + \frac{6656}{12825}k\hat{S}_3 + \frac{28651}{56437}k\hat{S}_4 - \frac{9}{5}k\hat{S}_5 + \frac{2}{55}k\hat{S}_6$$

$$\hat{I}_{t+1} = I_i + \frac{16}{135}k\hat{I}_1 + \frac{6656}{12825}k\hat{I}_3 + \frac{28651}{56437}k\hat{I}_4 - \frac{9}{5}k\hat{S}_5 + \frac{2}{55}k\hat{I}_6$$

$$\hat{R}_{t+1} = R_i + \frac{16}{135}k\hat{R}_1 + \frac{6656}{12825}k\hat{R}_3 + \frac{28651}{56437}k\hat{R}_4 - \frac{9}{5}k\hat{R}_5 + \frac{2}{55}k\hat{R}_6$$

With the following counting constants:

$$k\hat{S}_1 = \Delta t\left(\alpha - \beta\frac{I}{N}S - \mu S\right)$$

$$k\hat{I}_1 = \Delta t\left(\beta\frac{I}{N}S - \mu I - \nu I - mI\right)$$

$$k\hat{R}_1 = \Delta t(mI - \mu R)$$

$$k\hat{S}_2 = \Delta t(\alpha - \beta\left(\frac{I}{N} + kI_1\frac{1}{4}\right)\left(S_i + k\hat{S}_1\frac{1}{4}\right)\left(I_t + k\hat{I}_1\frac{1}{4}\right) - \mu(S_t + k\hat{S}_1\frac{1}{4})$$

$$k\hat{I}_2 = \Delta t(\beta\left(\frac{I}{N} + kI_1\frac{1}{4}\right)\left(S_t + k\hat{S}_1\frac{1}{4}\right)\left(I_t + k\hat{I}_1\frac{1}{4}\right) - \mu\left(I_t + k\hat{I}_1\frac{1}{4}\right) - \nu\left(I_t + k\hat{I}_1\frac{1}{4}\right) - m(I_t + k\hat{I}_1\frac{1}{4})$$

$$k\hat{R}_2 = \Delta t(m\left(I_t + k\hat{I}_1\frac{1}{4}\right) - \mu(R_t + k\hat{R}_1\frac{1}{4})$$

$$k\hat{S}_3 = \Delta t(\alpha - \beta\left(\frac{I}{N} + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right)\left(S_t + k\hat{S}_1\frac{3}{32} + k\hat{S}_2\frac{9}{32}\right)\left(I_t + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right) - \mu(S_t + k\hat{S}_1\frac{3}{32} + k\hat{S}_2\frac{9}{32})$$

$$k\hat{I}_3 = \Delta t(\beta\left(\frac{I}{N} + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right)\left(S_t + k\hat{S}_1\frac{3}{32} + k\hat{S}_2\frac{9}{32}\right)\left(I_1 + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right) - \mu\left(I_t + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right) - \nu\left(I_t + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right) - m(I_t + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32})$$

$$k\hat{R}_3 = \Delta t(m\left(I_t + k\hat{I}_1\frac{3}{32} + k\hat{I}_2\frac{9}{32}\right) - \mu(R_t + k\hat{R}_1\frac{3}{32} + k\hat{R}_2\frac{9}{32})$$

$$k\hat{S}_4 = \Delta t(\alpha - \beta\left(\frac{I}{N} + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right)\left(S_t + k\hat{S}_1\frac{3}{4} - \frac{3}{16}k\hat{S}_2 + \frac{9}{16}k\hat{S}_3\right)\left(I_t + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right) - \mu(S_t + k\hat{S}_1\frac{3}{4} - \frac{3}{16}k\hat{S}_2 + \frac{9}{16}k\hat{S}_3)$$

$$k\hat{I}_4 = \Delta t(\beta\left(\frac{I}{N} + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right)\left(S_t + k\hat{S}_1\frac{3}{4} - \frac{3}{16}k\hat{S}_2 + \frac{9}{16}k\hat{S}_3\right)\left(I_t + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right) - \mu\left(I_t + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right) - \nu\left(I_t + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right) - m(I_t + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3)$$

$$k\hat{R}_4 = \Delta t(m\left(I_t + k\hat{I}_1\frac{3}{4} - \frac{3}{16}k\hat{I}_2 + \frac{9}{16}k\hat{I}_3\right) - \mu(R_t + k\hat{R}_1\frac{3}{4} - \frac{3}{16}k\hat{R}_2 + \frac{9}{16}k\hat{R}_3)$$

$$k\hat{S}_5 = \Delta t(\alpha - \beta\left(\frac{I}{N} + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4\right)\left(S_t + k\hat{S}_1\frac{439}{216} - 8k\hat{S}_2 + \frac{3860}{513}k\hat{S}_3 - \frac{845}{4104}k\hat{S}_4\right)\left(I_t + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4\right) - \mu(S_t + k\hat{S}_1\frac{439}{216} - 8k\hat{S}_2 + \frac{3860}{513}k\hat{S}_3 - \frac{845}{4104}k\hat{S}_4)k\hat{I}_5$$

$$k\hat{I}_5 = \Delta t(\beta\left(\frac{I}{N} + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4\right)\left(S_t + k\hat{S}_1\frac{439}{216} - 8k\hat{S}_2 + \frac{3860}{513}k\hat{S}_3 - \frac{845}{4104}k\hat{S}_4\right)\left(I_1 + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4\right) - \mu\left(I_t + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4\right) - \nu\left(I_t + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4\right) - m(I_t + k\hat{I}_1\frac{439}{216} - 8k\hat{I}_2 + \frac{3860}{513}k\hat{I}_3 - \frac{845}{4104}k\hat{I}_4)$$

$$k\hat{S}_6 = \Delta t(\alpha - \beta\left(\frac{I}{N} - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right)\left(S_t - k\hat{S}_1\frac{8}{27} + 2k\hat{S}_2 - \frac{3544}{2565}k\hat{S}_3 - \frac{1859}{4104}k\hat{S}_4 - \frac{11}{40}k\hat{S}_5\right)\left(I_t - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right) - \mu(S_t - k\hat{S}_1\frac{8}{27} + 2k\hat{S}_2 - \frac{3544}{2565}k\hat{S}_3 - \frac{1859}{4104}k\hat{S}_4 - \frac{11}{40}k\hat{S}_5)$$

$$k\hat{I}_6 = \Delta t(\beta\left(\frac{I}{N} - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right)\left(S_t - k\hat{S}_1\frac{8}{27} + 2k\hat{S}_2 - \frac{3544}{2565}k\hat{S}_3 - \frac{1859}{4104}k\hat{S}_4 - \frac{11}{40}k\hat{S}_5\right)\left(I_1 - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right) - \mu\left(I_t - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right) - \nu\left(I_t - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right) - m(I_t - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5)$$

$$k\hat{R}_6 = \Delta t(m\left(I_t - k\hat{I}_1\frac{8}{27} + 2k\hat{I}_2 - \frac{3544}{2565}k\hat{I}_3 - \frac{1859}{4104}k\hat{I}_4 - \frac{11}{40}k\hat{I}_5\right) - \mu(R_t - k\hat{R}_1\frac{8}{27} + 2k\hat{R}_2 - \frac{3544}{2565}k\hat{R}_3 - \frac{1859}{4104}k\hat{R}_4 - \frac{11}{40}k\hat{R}_5)$$

Using MATLAB software, numerical solutions for the SIR mathematical model of tuberculosis spread were obtained using the *Runge Kutta Fehlberg* (RKF45) method. The results of 50 iterations can be seen in Table 3 below:

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Table 3. Results Analysis Of The Runge Kutta Fehlberg Method

Year	Iteration	S	I	R
2024	0	37451717,00	71550,00	17695,00
2025	1	37402495,16	53751,60	33050,62
2026	2	37353367,32	40372,12	44567,86
2027	3	37304326,10	30330,35	53185,93
2028	4	37255365,83	22791,80	59629,73
2029	5	37206482,11	17115,48	64456,07
2030	6	37157671,92	12862,45	68046,24
2031	7	37108932,71	9664,22	70720,32
2032	8	37060262,64	7257,95	72706,63
2033	9	37011660,37	5455,28	74168,80
2034	10	36963124,76	4097,61	75244,57
2035	11	36914655,00	3077,89	76026,94
2036	12	36866250,48	2313,49	76587,67
2037	13	36817910,66	1737,36	76985,01
2038	14	36769635,17	1305,30	77257,36
2039	15	36721423,67	980,97	77436,27
2040	16	36673275,91	736,64	77545,83
2041	17	36625191,68	553,58	77602,31
2042	18	36577170,78	415,91	77619,50
2043	19	36529213,07	312,35	77607,20
2044	20	36481318,41	234,76	77572,48
2045	21	36433486,67	176,33	77521,25
2046	22	36385717,74	132,45	77457,52
2047	23	36338011,50	99,55	77384,38
2048	24	36290367,87	74,76	77304,34
2049	25	36242786,75	56,17	77219,05
⋮	⋮	⋮	⋮	⋮
2073	49	35119370,82	0,06	74872,71
2074	50	35073325,13	0,04	74774,55

Source: Data processed by researchers

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It can be seen in table 3 that the numerical solution of the Runge Kutta Fehlberg method for the SIR model shows that the spread of tuberculosis has decreased from year to year. Based on Table 3, with the initial conditions $S_0 = 111.120.397, I_0 = 157.024, R_0 = 38.452$ using RKF45 $S_{50} = 35.073.325, I_{50} = 0{,}04, R_{50} = 74.774$. The infected subpopulation will gradually disappear to 0. Based on the table, it can be seen that in 2073 the number of infected individuals will be less than 0. This shows a promising reduction in infection rates for Indonesia if there is good cooperation from all authorities handling TB cases.

The graph of numerical iteration results using the *Runge Kutta Fehlberg* method for the spread of tuberculosis for each *Susceptible*, *Infected*, and *Recovered individual* using MATLAB *software* is shown in Figure 2.

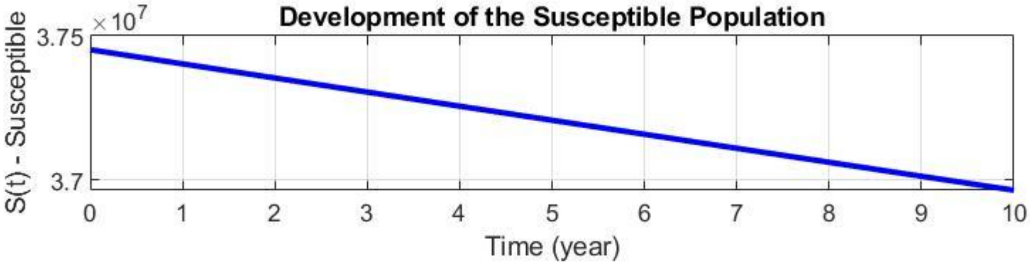

Figure 2. Development Of The Susceptible Population

Based on Figure 2, the susceptible population (blue line) decreases over time in the first 10 years, reaching 3.7, then gradually decreases until it reaches 0 in 50 years.

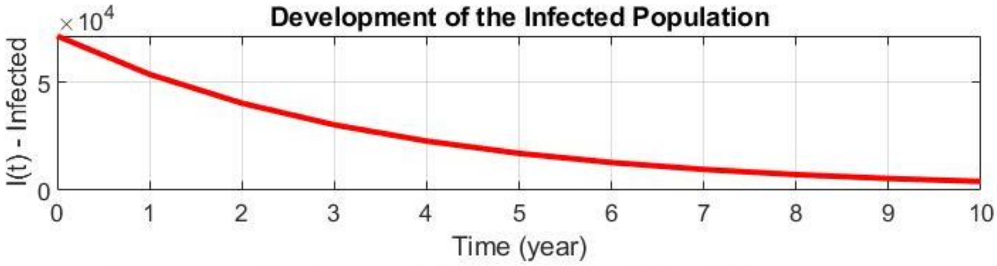

Figure 3. Development Of The Infected Population

Based on Figure 3, the infected population on the red line peaks at 0 years, indicating the infected population before iteration. Then the line decreases significantly to near 0 in the first 10 years. At 15 years, the line is already at 0 and continues to decline until the 50th year.

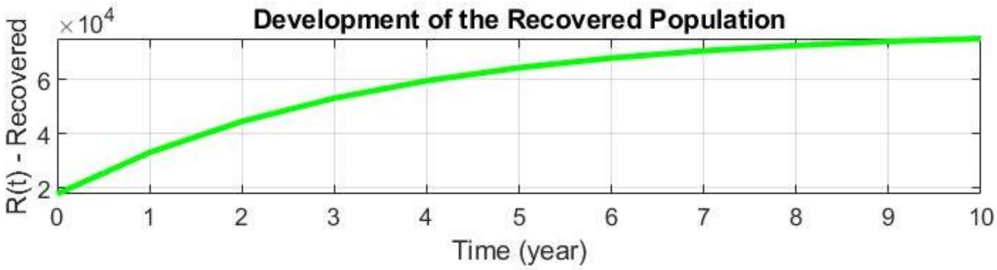

Figure 4. Development Of The Recovered Population

Based on Figure 4, which shows the development of the recovered population indicated by the green line. The recovered population line is at 0 at year 0, then increases in the first 15 years to its highest point, indicating that the recovered population dominates. At year 40, there is an insignificant change in number, namely a decrease. The decrease in the recovered population may be influenced by the number of deaths.

4. CONCLUSION

Based on the results of the Susceptible, Infected, Recovered (SIR) mathematical model for the spread of tuberculosis in Central Java, numerical results obtained using the Runge-Kutta-Fehlberg (RKF45) numerical method with a step size of $\Delta P = h = 1$ year and a time limit of 50 years indicate that the number of infected people is 104.126.462, less than one person is infected, and the number of recovered people is 162.079. Combining the RKF45 method to determine the numerical solution of the SIR mathematical model is still relevant for predicting the number of tuberculosis transmission cases over a long-time span.

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