The Measurement of Solar Limb Darkening at Hydrogen Alpha

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ABSTRACT

Sun's disc has a lower intensity at the limb, which looks darker than at the center. This effect is known as solar limb darkening. In this study, the Authors determine the solar limb darkening at hydrogen alpha with 656 nm thus we can compare the result to the previous studies. The authors have taken the solar images at Observatorium Ilmu Falak Universitas Muhammadiyah Sumatera Utara (OIF UMSU). The limb-darkening coefficient is obtained through least-square fitting with the linear limb-darkening law. The result shows that the linear limb-darkening coefficient is 0.607 and is in good agreement with the previous studies. In addition, this study is the initial stage of limb darkening research at OIF UMSU and can be continued with more observational data.

I. Introduction

The solar brightness is not uniform but decreases from the center of the disc towards the limb. This effect is called limb-darkening. Radiation seen at the center comes from the deeper layer where the temperature is higher than the radiation from the limb. Ref [1] was a pioneer in the determination of limb darkening followed by another researcher with various works. One of them was the measurement of the limb-darkening coefficient in various wavelengths. For instance, ref [2] measured limb darkening at infrared from 0.5 and 10.2-. Determination of the limb-darkening coefficient is important because it can explain the properties of a star's atmosphere (including the sun). Moreover, it also affects the star's angular diameter and stellar rotational velocity [3]. The direct measurement of limb darkening using the observational data can be easily carried out for the sun. Several studies for various wavelengths including [4] determine the limb darkening effect at 347 nm and 546 nm, [5] at 550 nm, [6] at 360 – 680 nm, in this work, we analyze solar limb darkening at hydrogen alpha (656 nm).

Hydrogen alpha is a narrow emission line with a wavelength of 656.28 nm. The wavelength exists in the red region in the visible spectral range. The observation made in this study captures the solar image in the hydrogen alpha line.

Furthermore, we also measured the limb-darkening coefficient using linear limb-darkening law derived by [1] (equation (1))

$$I(\mu) = I(1)(1 - u(1 - \mu)) \quad ...(1)$$

where $I(1)$ denote the specific intensity at the disc center and u is the linear limb-darkening coefficient. $\mu = \cos\theta$ where θ is the angle made by the line of sight and the normal to the surface at the observed point. Therefore, $\theta = 0°$ at the disc center and $\theta = 90°$ at the limb.

Several studies stated that the linear limb-darkening law may not be accurate. One of the studies is [7] that concludes the linear approximation is inaccurate for hot stars. For that reason, previous studies presented non-linear limb darkening, such as quadratic and logarithmic approximation [8]. However, for solar-type stars, the linear limb darkening law gives a good approximation [4] thus for this research we use the linear approximation.

II. Method

Solar limb darkening can be determined by a simple method, namely measuring the intensity along the solar disc from the center to the limb of the disc. The measurement of that intensity along the disc can be done easily using direct solar observations and processed with certain software.

Solar observation at hydrogen alpha (λ= 656 nm) uses imaging instruments consisting of a telescope and a charge-coupled device (CCD) detector. The specification of the telescope used in this study is the lunt solar system LS50THα telescope with a diameter of 50 mm and a focal length of 350 mm. This telescope is equipped with an etalon filter that works by canceling all wavelengths that reach the observer except for certain wavelengths.

In this type of telescope, the etalon has a wavelength of 656.28 nm (bandpass <0.7Å) thus it can observe the Sun in the hydrogen alpha (H-alpha) line. Etalon tuning with Doppler True Pressure Tuning system.

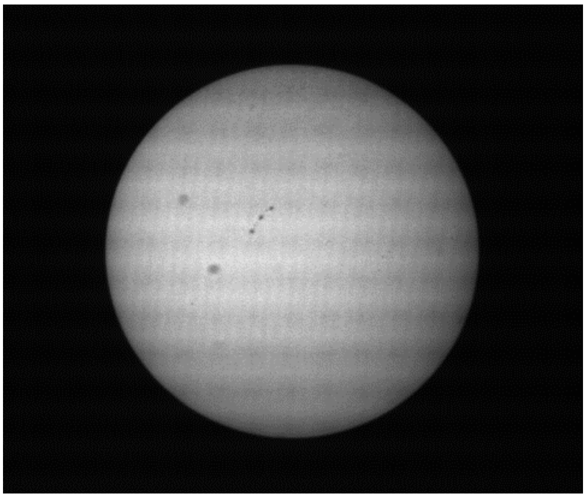

Figure 1. *The Solar disc that captured at OIF UMSU*

The type of CCD used is QHY 5L-II M with specifications shown in Table 1. With the pixel and image size as well as the size of the telescope, the observations provide a complete image of the solar disc (Figure 1). In the capturing process, the exposure is adjusted so we can get non-saturated images with a maximum exposure time of no more than 0.087 seconds.

Furthermore, the solar image obtained will be processed to analyze the limb-darkening effect using ImageJ software (Figure 2). ImageJ is a Java-based software for processing scientific image data.

Through imageJ, the intensity expressed in gray levels is calculated from the center to the limb. The intensity normalized towards the intensity at the disc center $I(\mu)/I(1)$. The distance from the center to the limb is expressed in pixels.

Equation (1) can be written into this form :

$$I\,(\mu)/I(1) = (1 - u(1 - \mu))$$

$$I\,(\mu)/I(1) = (1 - u + u\mu)$$

thus,

$$I\,(\mu)/I(1) = u\,\mu + (1 - u\,) \qquad \ldots(2)$$

Table 1. Specification of CCD QHY 5L-II M

Item	Specification
Resolution	1280 x 960
Pixel size	3.75 µm
Quantum Efficiency	74%

The relation between intensity $I\,(\mu)/\,I(1)$ and μ were obtained by least squares fitting. The formula used in this fitting is equation (2). The least squares method used was also carried out in [7], [8], and [9]. The coefficient of the fitting gives the value of the limb-darkening coefficient at wavelength 656 nm which can be compared with the results of previous studies.

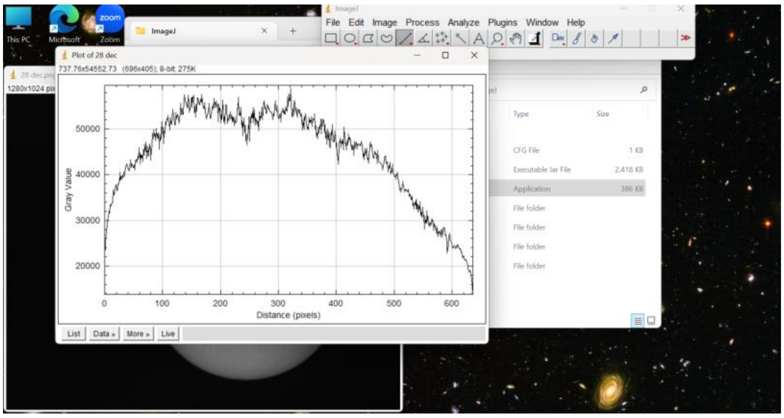

Figure 2. Analyze the limb darkening effect using ImageJ

III. Results and Discussion

Figure 3 shows the limb-darkening effect from the solar observation. The graph represents the variation of intensity from the disc center to the limb. The y-axis is the intensity at a certain θ which has been normalized towards intensity at the center. Meanwhile, θ becomes a variable that gives variation from the disc center to the limb where the disc center $\theta = 0°$ and the limb $\theta = 90°$.

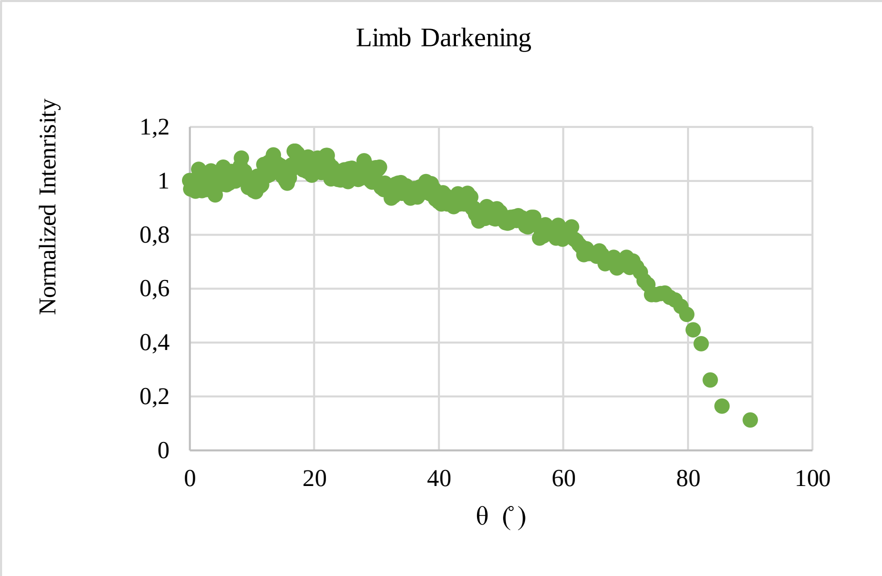

Figure 3. Limb darkening at $\lambda = 656\ nm$

From the graph Figure 3, the intensity is maximum at the center and continues to decrease. To understand the solar limb darkening at wavelength 656 nm, this study fits the observational data with the linear limb-darkening law as written in equation (2). The fitting between the linear approximation and the observational data using the least-square fitting method is shown in Figure 4.

The observational data in this study has good agreement with the linear limb-darkening law with R^2 = 0.8747. The value of 0.607 in the equation became the limb-darkening coefficient. The results obtained in this study were compared with the results in previous studies.

Table 2 displays the intensity (the intensity which has been normalized towards intensity at the center) at =1.0; 0.8; 0.6; 0.5; 0.4; 0.3; 0.2 to compare with previous studies. The number in Table 2 is compared with Allen's data [10] and A.S. Ramanathan [6]. Allen's data provide 184 limb darkening data for 24 wavelengths (200–2000 nm) at 9 points 0.8; 0.6 to 0.1; 0.05; 0.02. However, the data we are trying to compare in this study is Allen's data at a wavelength of 600 nm which is the closest wavelength to hydrogen alpha that is available in Allen's data (table 3). Whereas the data available on Ramanathan [6] (is the value of I (μ)/I(1) observed at the wavelength 360 – 667 nm.

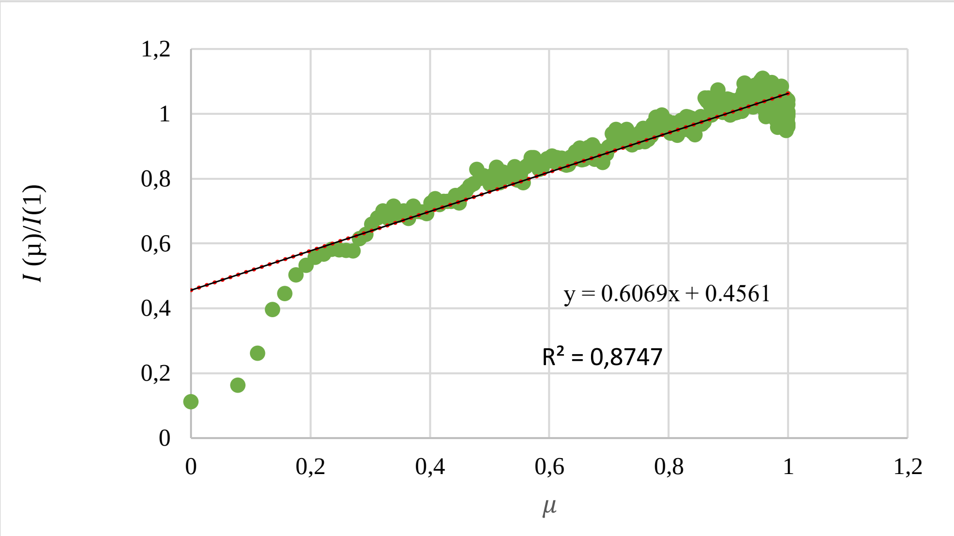

Figure 4. Plot of the intensity towards μ fitted with the linear limb-darkening law

Table 2. *I* (μ)/I(1) towards eight points radius from the center

μ	*I* (μ)/I(1)
1	1.0
0.8	0.967
0.6	0.869
0.5	0.782
0.4	0.725
0.3	0.659
0.2	0.557

Table 3. Comparison of limb darkening with Allen's data

μ	*I* (μ)/I(1)	Discrepancy
1	1	0
0.8	0.9	7.2%
0.6	0.788	9.8%
0.5	0.727	7.3%
0.4	0.664	8.8%
0.3	0.587	11.6%
0.2	0.508	9.2%

The result was compared with A.S. Ramanathan at a wavelength of 667 nm (table 4). From Table 3, the result of this research shows a discrepancy not greater than 12%, whereas with A.S. Ramanathan < 22% (table 4).

Table 4. Comparison limb darkening with A.S. Ramanathan

μ	I (μ)/I(1)	Discrepancy
1	1.0	0
0.8	0.942	2.6%
0.6	0.882	1.5%
0.5	0.862	9.8%
0.4	0.827	13.1%
0.3	0.771	15.7%
0.2	0.692	21.6%

The difference is quite large due to the different wavelengths used in determining the degree of limb darkening. For instance, Allen's data use wavelength at 600 nm, meanwhile in this study is 656 nm. However, the comparison of the limb-darkening coefficient between this study and previous studies shows a good agreement. Ref [11] obtained a limb-darkening coefficient of 0.61 (discrepancy = 0.5%) which is identical to the results in this study. Ref [4] acquired limb-darkening coefficients 0.837 and 0.631 for wavelengths λ = 347 nm and λ = 546 nm, respectively. The limb-darkening coefficient at λ = 546 nm fairly matches this study because the wavelengths are close (discrepancy = 3.8%). While at λ = 347 nm shows a discrepancy of 27% due to the difference of wavelengths.

The discrepancy arises because of the effect of wavelengths on the limb darkening. The degree of limb darkening decreases with increasing wavelength [8]. For the blue (shorter) wavelengths, the limb darkening will be stronger and will diminish at the red (longer) wavelengths as we can see in the comparison result above.

IV. Conclusion

The result of the limb-darkening coefficient at 656 nm from this study is 0.607. The graph of intensity against θ obtained from this study denotes how the intensity towards the limb is lower than in the center of the disc. The intensity was compared with Allen's data and A.S. Ramanathan shows discrepancies of < 12% and < 22%, respectively, due to differences in the wavelengths.

The limb-darkening coefficient is in good agreement with the values obtained from the previous study [11] which shows a discrepancy of 0.5%. In addition, the coefficient acquired from this study has lower results when compared to the coefficient at shorter wavelengths. Ref [4] shows the limb-darkening coefficient at λ = 347 nm greater than the limb-darkening coefficient from this study. This is because the limb darkening is more clearly visible at shorter wavelengths.

In addition, this study is the initial stage of limb darkening research at OIF UMSU and can be continued with more observational data.

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