AL-MARSHAD: JURNAL ASTRONOMI ISLAM DAN ILMU-ILMU BERKAITAN
ISSN 2442-5729 (print) || ISSN 2598-2559 (online)
http://jurnal.umsu.ac.id/index.php/almarshad
DOI: https://doi.org/10.30596/jam.v10i1.16616 || Vol. 10, No. 1 Juni 2024

Lunar Position Calculation Algorithm With Truncated ELP/MPP02 Series

Yusrifal Fais Abdillah[1*], Muhammad Muadz Dzulikrom[2]
[1]Universitas Islam Negeri Walisongo
(Jl. Walisongo No.3-5, Tambakaji, Kec. Ngaliyan, Semarang, Indonesia)
[2]Universitas Islam Negeri Sunan Ampel
(Jl. Ahmad Yani No.117, Jemur Wonosari, Kec. Wonocolo, Surabaya, Indonesia)
[*]Email: faizyusrifal4@gmail.com

Abstract

The truncated ELP/MPP02 series is a series of lunar position calculations based on the ELP/MPP02 theory developed by Chapront and Francou in 2002. This series reduces the lunar correction data, reducing the number of corrections from 35,901 to 253 data points, which are used to calculate the ecliptic longitude, ecliptic latitude and distance of the Moon from the Earth. The conclusions of this research are as follows: First, the calculation of lunar coordinates with the truncated ELP/MPP02 series involves the calculation of ecliptic longitude (L), ecliptic latitude (B), and the distance of the Moon from Earth (R) using the provided periodic terms, and then correcting for $\Delta\lambda$ and $\Delta\psi$ to calculate the apparent position of the Moon. Secondly, the truncated ELP/MPP02 series has a maximum error of 5" for L, 1.26" for B and 2.43 km for R in the time range 0 AD to 3000 AD. In the case of L, B and R, these errors do not exceed 5', 1' and 14 km respectively when calculations are made between 3000 BC and 1000 BC. For lunar and solar eclipses, the series shows an accuracy of 5.8 seconds for lunar eclipses in the time range 2000 AD to 2050 AD, and 4 seconds for solar eclipses.

Keywords: ***Algorithm, Lunar Position Calculation, Truncated ELP/MPP02 Series***

Article Info
Received:
13 September 2023
Revised:
17 November 2023
Accepted:
28 Desember 2023
Published:
29 Juni 2024

A. Introduction

An accurate algorithm for calculating the position of the lunar surface plays an important role in various aspects of life, from astronomy to practical applications in navigation, calendars and religion. Lunar position algorithms in the study of astrology are needed to calculate lunar eclipses, solar eclipses, and to determine the beginning of the lunar month through crescent observations.The term ELP stands for Ephemeride Lunaire Parisienne, a theory of lunar position developed by Jean Chapront, Michelle Chapront-Touzé and several other researchers at the Bureau des Longitudes. The Bureau des Longitudes was a scientific institution founded in France in the 19th century[1]. MPP02 is then one of a series of several existing lunar position calculation models, such as 2000-82[2], 2000-85[3], and the JANUS series[4].

ELP/MPP02 is a semi-analytical solution for lunar motion developed by Jean Chapront and Gerard Francou in 2002. ELP/MPP02 is a series of improvements to its predecessors, the ELP2000-82 and ELP2000-85 theories. The main article discussing the ELP/MPP02 theory is entitled "The Lunar Theory ELP Revisited". Introduction of New Planetary Perturbations", published in the journal Astronomy and Astrophysics in 2003[5].The lunar position theory in ELP/MPP02 contains a total of 35,901 data corrections, with maximum errors compared to the JPL DE405 and DE406 series of 2.4" in ecliptic longitude, 0.5" in ecliptic latitude, and 1.4 km in distance[5].

In order to improve the efficiency of the lunar position calculation, we truncated the existing data in the ELP/MPP02 theory. The original data of the lunar position correction in ELP/MPP02 theory includes 35,901 corrections, in this study the researchers reduced the data to about 253 corrections by including corrections from planetary perturbations such as Mercury, Venus, Earth, Mars, Jupiter, Saturn. The lunar position data used in this study are the result of an adjustment of ELP/MPP02 with JPL-DE405. JPL-DE405 (Jet Propulsion Laboratory Development Ephemeris 405).The purpose of this data truncation is to improve the computational efficiency of the lunar position calculation. Using the full data set of 35,901 corrections can result in slow computational performance, so the researchers chose this approach in the belief that the truncated data would still provide sufficient accuracy to produce an accurate lunar position.

The purpose of this study is to determine the algorithm for calculating the lunar position using the truncated ELP/MPP02 series and to determine the accuracy of the truncated

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ELP/MPP02 series. The benefit of this research is to contribute to science in the form of developing an accurate algorithm for calculating the position of the Moon by adopting the latest set of data based on the theory of ELP/MPP02 corrected with JPL-DE405. Remember that the lunar position calculation developed by Jean Meeus is still based on the ELP2000-82 theory and does not include some corrections for interference from several other planets.

B. Methods

This is a library research. The primary sources used are Chapront and Francou's journal "Lunar Theory ELP Revisited. Introduction of New Planetary Perturbations" and the ELP/MPP02 database at ftp://polac.obspm.fr/pub/2_lunar_solutions/2_elpmpp02. The data were analysed using a qualitative descriptive approach to answer the research questions relating to the algorithm for calculating the lunar position using the ELP/MPP02 truncated series and the accuracy of the ELP/MPP02 truncated series.

C. Resultsand Discussion

Results

ELP/MPP02 Truncated Series Lunar Coordinate Display

The calculation of the position of the Lunar in this study is based on the position of the Lunar in ecliptic coordinates, by calculating the ecliptic longitude (*L*), the ecliptic latitude (*B*) and the distance of the Moon from the centre of the Earth (*R*). *L* and *B* refer to *the mean dynamical equinox of the date*, measured in degrees of arc, while the value of R is measured in kilometres.

The coordinate representations of *L, B, R,* in this study are derived from basic determination values such as *D, F, l, l', Me, Ve, Ea, Ma, Ju, Sa*. *D* is the mean elongation of the Lunar from the Sun. *F* is the difference between the mean longitude of the Lunar and the mean longitude of the ascending node of the Lunar. l is the mean anomaly of the Lunar. l' is the mean anomaly of the Sun. Me, Ve, Ea, Ma, Ju, Sa are the mean heliocentric longitudes of the planets Mercury, Venus, Earth, Mars, Jupiter, Saturn. D, F, l, l' refer to the mean dynamic equinox of the date, while Me, Ve, Ea, Ma, Ju, Sa refer to the mean ecliptic and equinox of J2000. The values of the planetary arguments are calculated on the basis of the VSOP2000 theory[6].

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The expressions of the basic arguments are given in the time argument function T. These basic arguments are expressed in units of degrees, as shown below:

D = 297.850207531 + 445267.111412406 T – 18.9600833 $10^{-4}T^2$+ 1.7795417 $10^{-6}T^3$–1.1689 $10^{-6}T^4$

F = 93.272099894 + 483202.017490680 T –36.67283139 $10^{-4}T^2$– 0.30625833 $10^{-6}T^3$– 0.168611 $10^{-6}T^4$

l = 134.96333415 + 477198.867497725 T + 87.25712217 $10^{-4}T^2$ + 14.365133 $10^{-6}T^3$-7.081667 $10^{-6}T^4$

l' = 357.529080097 + 35999.050293583 T – 1.526291667 $10^{-4}T^2$ + 0.03531667 $10^{-6}T^3$– 0.31191667 $10^{-6}T^4$

Me = 252.250893589 + 149474.071518076 T

Ve = 181.979099561 + 58519.212570546 T

Ea = 100.466427458 + 36000.769746999 T

Ma = 355.434345216 + 19141.696237654 T

Ju = 34.351494276 +3036.302602651 T

Sa = 50.077472915 +1223.510592365 T

ELP/MPP02 Truncated Series Lunar Coordinate Calculation

The ecliptic longitude correction table can be found in Appendices 1 and 2, the correction is used to calculate the ecliptic longitude of the Lunar in formula (1). Both the$C_L$andC'_Lcalculations are computed for each value of$n$ from 1 to the maximum value of$n$. The coefficientsL_n,$\lambda_n^{(0)}$, $\lambda_n^{(1)}$, $\lambda_n^{(2)}$, $\lambda_n^{(3)}$, $\lambda_n^{(4)}$in Appendix 1 are used to calculate the $C_L$value,and the coefficientL'_n,$\lambda'^{(0)}_n$, $\lambda'^{(1)}_n$, $\lambda'^{(2)}_n$, $\lambda'^{(3)}_n$, $\lambda'^{(4)}_n$in Appendix 2 are used to calculate theC'_Lvalue.

The correction table for the lunar ecliptic latitude can be found in Appendix 3, the correction is used to calculate the lunar ecliptic latitude in formula (2). TheC_B calculation is performed for each value of$n$ between 1 and the maximum value of$n$.The coefficientsB_n, $\beta_n^{(0)}$, $\beta_n^{(1)}$, $\beta_n^{(2)}$, $\beta_n^{(3)}$, $\beta_n^{(4)}$in Appendix 3 are needed to calculate the$C_B$value, so that the value of the lunar ecliptic latitude can be known from the summation of the$C_B$correction.

The correction table for the distance of the Lunar from the centre of the Earth can be found in Appendices 4 and 5, the correction is used to calculate the distance of the Lunar from the centre of the Earth in formula (3). The calculation$C_R$or$C'_R$is carried out for each value of$n$ between 1 and the

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maximum value of$n$. The coefficientsR_n, $\alpha_n^{(0)}$, $\alpha_n^{(1)}$, $\alpha_n^{(2)}$, $\alpha_n^{(3)}$, $\alpha_n^{(4)}$in Appendix 4 are used to calculate theC_R, value and the coefficientsR'_n, $\alpha'^{(0)}_n$, $\alpha'^{(1)}_n$, $\alpha'^{(2)}_n$, $\alpha'^{(3)}_n$, $\alpha'^{(4)}_n$in Appendix 5 are used to calculate$C'_{R.}$ value.

To calculate the coordinates of the Moon in this paper, first determine the time T, measured in Julian centuries with a span of 36525 ephemeris days from epoch J2000 (1 January 2000 at 12:00):

$$T = \frac{JDE - 2451545}{36525} \tag{1}$$

Then, the longitude of the Lunar$L$ is expressed with a value:

$$L = 218.316634897222 + 481267.881161438\,T - 0.00159455422\,T^2 + T^3 / 554273 - T^4 / 54722058 + C_L + (C'_L\ T) \tag{2}$$

with:

$$C_L = L_n \sin(\lambda^{(0)}{}_n + \lambda^{(1)}{}_n T + \lambda^{(2)}{}_n 10^{-4} T^2 + \lambda^{(3)}{}_n 10^{-6} T^3 + \lambda^{(4)}{}_n 10^{-8} T^4)$$
$$C'_L = L'_n \sin(\lambda'^{(0)}{}_n + \lambda'^{(1)}{}_n T + \lambda'^{(2)}{}_n 10^{-4} T^2 + \lambda'^{(3)}{}_n 10^{-6} T^3 + \lambda'^{(4)}{}_n 10^{-8} T^4)$$

The latitude of Lunar B is expressed by the value:

$$B = C_B \tag{3}$$

with:

$$C_B = B_n \sin(\beta^{(0)}{}_n + \beta^{(1)}{}_n T + \beta^{(2)}{}_n 10^{-4} T^2 + \beta^{(3)}{}_n 10^{-6} T^3 + \beta^{(4)}{}_n 10^{-8} T^4)$$

The lunar distance R is expressed by:

$$R = 385000.509715468 + C_R + C'_R\,T \tag{4}$$

with:

$$C_R = R_n \cos(\alpha^{(0)}_n + \alpha^{(1)}_n T + \alpha^{(2)}_n 10^{-4} T^2 + \alpha^{(3)}_n 10^{-6} T^3 + \alpha^{(4)}_n 10^{-8} T^4)$$
$$C'_R = R'_n \cos(\alpha'^{(0)}_n + \alpha'^{(1)}_n T + \alpha'^{(2)}_n 10^{-4} T^2 + \alpha'^{(3)}_n 10^{-6} T^3 + \alpha'^{(4)}_n 10^{-8} T^4)$$

The correction calculations for L, B, R are calculated by summing the data quantities in Appendices 1 to 5 according to the procedure described above. The letter C refers to the term correction.

For the Moon's ecliptic longitude and distance, correction (C) is required to obtain the Lunar's ecliptic longitude and true distance by summing the mean data with the sum of the correction terms. Whilst the ecliptic longitude of the Lunar, the sum of the corrections has produced the final value of the true ecliptic longitude of the lunar.

In calculating the lunar longitude, the periodic correction term C_L from the first row is summed to 108 ($\sum L_n$). ThenC'_L , is summed from the first row to row 6 ($\sum L'_n$). This also applies to the calculation of ecliptic latitude and lunar distance.

To calculate the apparent position of the lunar surface as seen by an observer on Earth, severalcorrections must be made, these are the aberration and nutation corrections. Aberration is a correction for the motion of the Earth relative to the Sun, which causes celestial bodies observed from the Earth to appear slightly misaligned in the sky[7]. In the case of the position of the lunar, the aberration value in the ecliptic longitude of the lunar can be applied, the aberration formula in the ecliptic longitude of the lunar in degrees, accompanied by the correction of the speed of light according to Clemence et al, can be calculated as follows:

$$\Delta\lambda = -0.0001953 - 0.0000106\sin(224.96333 + 477198.86750\,T + 0.0087257\,T^2 + T^3 / 69613 \qquad (5)$$

After calculating the aberration correction, calculate the nutation correction in longitude ($\Delta\psi$). This nutation correction is needed to correct the Moon's ecliptic longitude position based on the mean equinox reference to the true equinox[8]. Thus, the Moon's apparent ecliptic longitude coordinates can be calculated by $\lambda_{\text{Evident}} = L + \Delta\lambda + \Delta\psi$. In addition to the longitude nutation, the ecliptic tilt or obliquity is also required. The value of the ecliptic obliquity is calculated by summing the mean value of the ecliptic obliquity (ε_0)with the nutation in the ecliptic obliquity ($\Delta\varepsilon$), giving the apparent true ecliptic obliquity (ε) on the

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DOI: https://doi.org/10.30596/jam.v10i1.16616 || Vol. 10, No. 1 Juni 2024

calculated date.

For the latest high-precision calculations of nutation in longitude or ecliptic tilt, it is recommended to use the IAU 2000/2006 standard, described in Petit and Luzum[9]. If an accuracy of 1" for $\Delta\psi$ and 0.4" for $\Delta\varepsilon$ is considered sufficient for nutation values based on IAU 2000/2006, the following formula can be applied:

$$\Delta\psi = (17''.2064245 + 0''.0174188\,T)\sin(305.0334378 - 1934.1362620\,T + 0.0020756\,T^2 + T^3/467411 - T^4/60616265 \quad (6)$$

$$\Delta\varepsilon = (9''.2052332 + 0''.000883\,T)\cos(125.0349840 - 1934.1362620\,T + 0.0020756\,T^2 + T^3/467411 - T^4/60616265 \quad (7)$$

with the mean ecliptic inclination (ε_0) based on IAU 2000/2006[10]:

$$\varepsilon_0 = 23.43927944 - 0.013010214\,T - T^2/19661387 + T^3/1796945 - T^4/6250000000 - T^5/82949308756 \quad (8)$$

In equatorial coordinates, the apparent position of the lunar object can be calculated as follows[11]:

$$\tan\alpha = \frac{\sin\lambda\cos\varepsilon - \tan\beta\sin\varepsilon}{\cos\lambda} \quad (9)$$

$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda \quad (10)$$

with:

$\lambda \quad = L + \Delta\lambda + \Delta\psi$

$\varepsilon \quad = \varepsilon_0 + \Delta\varepsilon$

For example, here the author calculates the position of the lunar nodes with respect to the mean dynamical equinox of the date based on the ELP/MPP02 truncated series on 11 September 2023 at 0^h TD:

JD = 2460198.5

T = 0.236 919 917 86

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C_L(°) = 3.399 299 046

C'_L (°) = – 0.000 268 07

L = 123° 39' 46"

B = 05° 06' 57"

C_R(km) = 20110.818 407 3

C'_R(km) = 0.036 616 9

R (km) = 405111.336 8

Then calculate the aberration correction using formula (5) and the nutation correction using the IAU 2000/2006 standard full correction, giving

$\Delta\lambda$ = -0".669115454

$\Delta\psi$ = -6".954680478

This is how the longitude of the ecliptic of the lunar appears to be:

Λ_{Evident} = 123° 39' 38"

Based on the JPL-DE441 ephemeris, the values of apparent ecliptic longitude, ecliptic latitude and lunar distance are as follows[12]:

Λ_{Evident} = 123° 39 '37"

B = 05° 06' 55"

R = 405109.77115

Discussion

Validity and accuracy of ELP/MPP02 truncated series lunar coordinate calculations

Here the author compares the results of the calculations with the results of NASA's JPL-DE441 ephemeris, and produces the following average differences in ecliptic longitude and latitude, and distance in kilometres:

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Table 1. Accuracy of ELP/MPP02 Truncated Lunar Series with JPL-DE441

Periode	Difference		
	Longitude	Latitude	Distance
3000 SM – 2000 SM	4.72'	27.92"	13.76
2000 SM – 1000 SM	2.31'	13.47"	7.31
1000 SM – 0 M	55.21"	5.41"	4.2
0 M – 1500 M	12.04"	1.74"	2.72
1500 M – 1900 M	2.76"	1.16"	2.37
1900 M – 2100 M	2.22"	1.13"	2.35
2100 M – 2500 M	2.63"	1.15"	2.35
2500 M – 3000 M	4.87"	1.26"	2.43

The author carried out an accuracy test using the RMSE (Root Mean Square Error) model. From Table 1 it can be seen that the accuracy of the ELP/MPP02 truncated series data has good accuracy when used in the years 0 AD - 3000 AD. Here the author also tests the accuracy of the Jean Meeus version of the lunar data with NASA's JPL-DE441 ephemeris, and when tested using RMSE (Root Mean Square Error) produces the following accuracy in ecliptic longitude, ecliptic latitude and distance in kilometres:

Table 2. Accuracy of Jean Meeus Version of Lunar Data with JPL-DE441

Periode	Differences		
	Longitude	Latitude	Distance
3000 SM – 2000 SM	37.74'	2.57'	164.6
2000 SM – 1000 SM	22.18'	1.49'	97.63
1000 SM – 0 M	11.08'	44.27"	49.65
0 M – 1500 M	3.34'	13.38"	15.3
1500 M – 1900 M	10.06"	1.25"	3.15
1900 M – 2100 M	2.77"	1.02"	2.95
2100 M – 2500 M	13.4"	1.29"	3.11
2500 M – 3000 M	1.05'	3.85"	5.85

It can be seen from Table 2 that the accuracy of Jean Meeus' lunar data is less accurate when calculations are made before 1500 AD, with errors increasing every 1000 years before 0 AD. In contrast to the ELP/MPP02 truncated series, the maximum error when calculations are made before 0 AD to 3000 BC is no more than 5' for ecliptic longitude, 30" for ecliptic latitude, and 15 km for distance. On the other hand, for the Jean Meeus data, the maximum error for calculations from 0 AD to 3000 BC is 38' for ecliptic longitude, 3' for ecliptic latitude, and 165 km for distance.

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Knowing that the ELP/MPP02 truncated series lunar data have good accuracy, the author tries to use the ELP/MPP02 truncated series lunar data to perform lunar and solar eclipse calculations and to validate with eclipse calculations using the JPL-DE441 ephemeris.The author compares it with the JPL-DE441 ephemeris version[13] considering that the JPL ephemeris developed by NASA's Jet Propulsion Laboratory (JPL) is made using accurate measurement data based on years of direct observation[14].

Calculation of solar and lunar eclipses as a comparison with the JPL-DE441 ephemeris, the solar data used by the author are calculated using the VSOP2000 theory developed by Moisson and Bretagnon[6], while the lunar data are calculated using the ELP/MPP02 truncated series. The nutation correction is calculated according to the IAU 2000/2006 standard by applying the full periodic correction.

The lunar and solar eclipse calculations presented in Tables 3 and 4 have been calculated using the Explanatory Supplement to The Astronomical Almanac by E. Urban and Seidelmann[7]. Comparative data on the results of lunar and solar eclipses are calculated with reference to JPL-DE441 ephemeris data taken from the website http://ytliu.epizy.com/eclipse/, as both calculations are calculated using the Explanatory Supplement to The Astronomical Almanac book. This book is used by both NASA and the Nautical Almanac UK as a reference for calculating lunar and solar eclipses. Sampling of lunar eclipses is random, with lunar eclipse intervals occurring every 5 years from 2000 to 2050:

Table 3. Difference in lunar eclipse calculation results between VSOP2000 ephemeris & ELP/MPP02 truncated series with JPL-DE441 ephemeris (difference in seconds)

Date	Type	Average difference of each phase
21/1/2000	Total	4
16/7/2000	Total	0
24/4/2005	Penumbra	6
17/10/2005	Partial	4
26/6/2010	Partial	6
21/12/2010	Total	4
4/4/2015	Total	16
28/9/2015	Total	5
10/1/2020	Penumbra	4
5/6/2020	Penumbra	3
5/7/2020	Penumbra	2

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Date	Type	Average difference of each phase
30/11/2020	Penumbra	4
14/3/2025	Total	6
7/9/2025	Total	3
15/6/2030	Parsial	3
9/12/2030	Penumbra	3
22/2/2035	Penumbra	7
19/8/2035	Partial	11
26/5/2040	Total	7
18/11/2040	Total	3
3/3/2045	Penumbra	5
27/8/2045	Penumbra	1
6/5/2050	Total	6
30/10/2050	Total	3

The author analyses the accuracy of the lunar eclipse calculation results from Table 3 using RMSE (Root Mean Square Error), so it is known that the accuracy of the VSOP2000 ephemeris & ELP/MPP02 truncated series is 5.8 seconds.For the results of the solar eclipse calculation can be seen in Table 4, the calculation of the solar eclipse calculated here is a general solar eclipse or solar eclipse in global coverage. The sampling of eclipses is random, with eclipse intervals occurring every 5 years, calculated from 2000 to 2050:

Table 4. Difference in eclipse calculation results between VSOP2000 ephemeris & ELP/MPP02 truncated series with JPL-DE441 ephemeris (difference in seconds of time)

Date	Type	Average difference of each phase
5/2/2000	Parsial	1
1/7/2000	Parsial	0
31/7/2000	Parsial	1
25/12/2000	Parsial	1
8/4/2005	Hibrida	2
3/10/2005	Cincin	8
15/1/2010	Cincin	1
11/7/2010	Total	2
20/3/2015	Total	7
13/9/2015	Parsial	2
21/6/2020	Cincin	4
14/12/2020	Total	1
29/3/2025	Parsial	2
21/9/2025	Parsial	1

1/6/2030	Cincin	1
25/11/2030	Total	8
9/3/2035	Cincin	8
2/9/2035	Total	8
11/5/2040	Parsial	3
4/11/2040	Parsial	1
17/2/2045	Cincin	1
12/8/2045	Total	2
20/5/2050	Hibrida	5
14/11/2050	Parsial	2

The author analyses the accuracy of the global eclipse calculation results from Table 4 using the RMSE (Root Mean Square Error), so the accuracy of the lunar data according to the ELP/MPP02 truncated series is 4 seconds.

D. Conclusion

The calculation of the lunar coordinates with the ELP/MPP02 truncated series is based on the position of the Lunar in the ecliptic coordinates by calculating the ecliptic longitude (L), the ecliptic latitude (B) and the distance of the Lunar from the centre of the Earth (R). Each calculation of L, B, R is made using the periodic correction terms given in Appendices 1 to 5. L can be calculated from formula (2) by summing theC_L andC'_Lcorrections. B can be calculated from formula (3) by summing allC_B corrections in Appendix 3. R can be calculated using formula (4) by summing the C_R andC'_R in Appendices 4 and 5. Then, when calculating the apparent position of the lunar as observed from Earth, the aberration correction in ecliptic longitude is added as in formula 5, and the nutation in ecliptic longitude can be accurately calculated using the IAU 2000/2006 standard.

The accuracy of the lunar coordinates with the ELP/MPP02 truncated series when tested with the RMSE (Root Mean Square Error) model, with comparison of lunar position data based on the JPL-DE441 ephemeris, gives good values. For the calculation range from 0 M to 3000 M with a maximum error of 5" for ecliptic longitude, 1.26" for ecliptic latitude and 2.43 Km for the distance of the Moon from the centre of the Earth. In the case of ecliptic longitude, latitude and distance, these errors increased by no more than 5', 1' and 14 Km respectively when calculations were made in the years 3000 BC to 1000 BC. Accuracy tests are also used to predict lunar and solar eclipses by comparing the VSOP2000 ephemeris & ELP/MPP02 truncated series with the JPL-DE441 ephemeris, in the case of lunar eclipses for the range 2000 AD to 2050 AD by sampling every 5 years the accuracy of the ELP/MPP02 truncated series Moon data & VSOP2000 Sun data has an accuracy of 5.8 seconds of time, for solar eclipses it has an accuracy of 4 seconds of time.

The suggestion from this research is that the use of ELP/MPP02 truncated series lunar data can be a good choice for calculating lunar coordinates in the time range 3000 BC to 3000 AD, especially if the error received is still within acceptable tolerance. These calculations are very useful for calculating the position of the new moon, for calculating Hisab-based calendars, and for calculating solar and lunar eclipses in the next few years. The ELP/MPP02 truncated lunar data series with 253 corrections can be used in computer programs because the amount of data corrections is not so large and the calculation is simple, with a sufficient level of accuracy.

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AL-MARSHAD: JURNAL ASTRONOMI ISLAM DAN ILMU-ILMU BERKAITAN
ISSN 2442-5729 (print) || ISSN 2598-2559 (online)
http://jurnal.umsu.ac.id/index.php/almarshad
DOI: https://doi.org/10.30596/jam.v10i1.16616 || Vol. 10, No. 1 Juni 2024

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