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Mathematical Analysis of 5° + 8° Parameters in the Unified Global Hijri Calendar

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Abstract

This research departs from the need to strengthen the scientific basis of the criteria of the Unified Global Hijri Calendar (KHGT), especially the criteria for the height of the hilal 5° and elongation of 8° guided by Muhammadiyah. So far, official documents on the origin of these criteria have not been widely publicized, raising questions among astronomers, astronomers, and religious institutions. The main objective of the study was to explain the mathematical analysis behind the selection of the 5° + 8° parameters. The research method was carried out with a theoretical approach using basic astronomical equations to analyze the moon's height, elongation, and azimuth difference, as well as assess its correlation with the possible visibility of the hilal. The results of this study show that the minimum hilal height of 5° and the minimum elongation of 8° have a strong scientific foundation, and do not appear arbitrarily, but are closely related to Bruin's theoretical analysis in 1977 which mapped the relationship between the height of the Moon, ARCV, the depression of the Sun, the width of the moon while the elongation parameter of 8° has a strong correlation with the theory of the visibility limit of the hilal proposed by André Danjon in 1936 the determination of 8° elongation in the KHGT can be It is understood as a form of scientific refinement that combines the theoretical limits of Danjon with an approach to optimal visibility of Bruin's analysis.

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A. Introduction

The Hijri calendar is a very important calendar system in the lives of Muslims, because it realizes the unity of the ummah with a globally integrated calendar and minimizes differences between countries in the implementation of worship based on the determination of the beginning of the Hijri month [1], [2], [3] The Global Islamic Calendar is an urgent need for Muslims today [4]. However, until now, there has been no global agreement on a Unified Hijri calendar. One of the main factors causing this is the difference in the visibility criteria of the new moon used by various Islamic countries and organizations.

Muhammadiyah as one of the largest Islamic organizations in Indonesia since

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Muharram 1447 H has used the criteria of a minimum hilal height of 5° and an elongation of at least 8° in determining the beginning of the Hijri month [5]. Previously, Muhammadiyah used the criteria of Hisab Wujudul Hilal. However, official documents explaining the mathematically and geometrically origin of the 5° + 8° parameter are still rarely published openly. This raises questions, both among astronomers and astronomers, regarding the scientific legitimacy of the criteria.

From an astronomical point of view, the 5° + 8° parameter contains two important aspects: (1) the height of the moon above the horizon at sunset/Arc of Vision (ARCV), and (2) the angular distance (elongation) of the moon-sun or Arc of Light (ARCL). Both are the main indicators of the possibility of seeing the new moon. However, the mathematical-geometric relationship between these parameters and the visibility of the hilal still requires in-depth study.

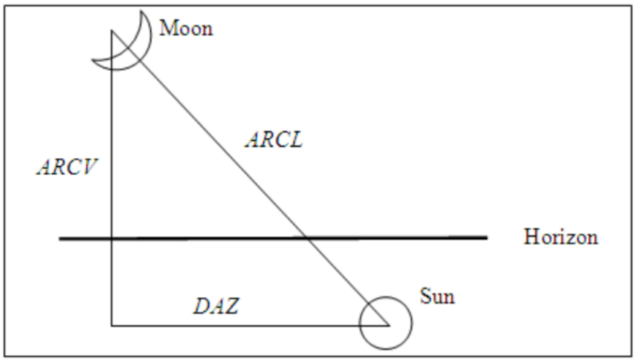

Figure 1. ARCV and ARCL

To answer the formulation of this research problem, a combination approach between theoretical studies and empirical analysis is used. From the theoretical side, the study examined the configuration of the position of the moon, sun, and earth with a focus on the parameters of elongation, moon height, and azimuth difference. Through basic astronomical equations, calculations are made regarding the height of the hilal and the elongation at sunset. This analysis was then directed to identify the mathematical correlation between the altitude of 5° and the elongation of 8° with the chance of visibility of the hilal based on the optical geometry conditions of the atmosphere.

This theoretical study is also enriched by an in-depth examination of the theory of lunar visibility that has been put forward by Danjon and Bruin. Danjon's theory provides a minimum limit of elongation for the moon to be observed, while the Bruin theory outlines the photometric and atmospheric aspects that affect the intensity of the moon light. The selection of the study of these two visibility theories is inseparable from historical factors,

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namely the determination of the 5° + 8° criterion which was first carried out in 1978 in Turkey, so that the study of visibility theories before 1978, namely Danjon Theory and Bruin Theory, is suspected to strongly influence these parameters. By examining these two theories in depth, the study sought to test the extent to which the 5° + 8° parameter is consistent with the visibility foundation that has been tested in the astronomical literature.

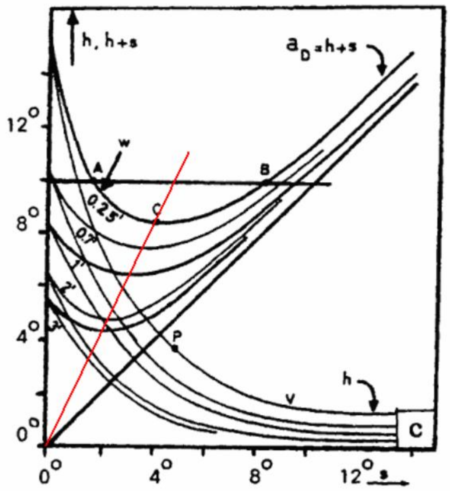

Figure 2. Bruin's Visibility Theory

In Figure 2. Bruin developed a theoretical graph in 1977, which mapped the Moon's altitude and ARCV versus the Sun's depression using various crescent moon width values. [6].

Furthermore, an empirical approach is carried out through the collection of rukyat hilal data from various credible sources, such as ICOP (Islamic Crescents' Observation Project), MABIMS, international astronomical publications and direct observation using telescopes. This observation data was analyzed to mark cases of visible or invisible hilal in conditions close to the parameter of 5° + 8°. Statistical tests in the form of accuracy, sensitivity, and specificity are used to assess the validity of these criteria on field data.

With a combination of mathematical-geometric approaches, global empirical data analysis, and enrichment of classical visibility theory, this research is expected to be able to produce a solid scientific foundation for the application of the 5° + 8° criteria. This result is important in strengthening the astronomical argument while making a real contribution to the discourse on the unification of the Unified Global Hijri Calendar (KHGT).

The study of the visibility of the moon has long been a concern for scientists, with various theories and criteria born over time [6] Danjon (1936), for example, established that the moon is impossible to see if its elongation is less than 7°, although this criterion is

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considered too general because it does not take into account the height of the moon [7]. Ilyas then tried to offer a visibility limit based on astronomical data, but did not set a standard number for the combination of height and elongation [8]. Yallop further introduced a global data-driven visibility probability model, generating complex curves that describe the visibility level of the hilal. Meanwhile, Odeh developed the Crescent Visibility Index (CVI), which practically classifies the conditions of the new moon in the categories of definitely visible, probably visible, or impossible to see [2] At the regional level, MABIMS (2021) updated the criteria to be 3° high and 6.4° elongation as an attempt to compromise between the astronomical base and the need for calendar unification in Southeast Asia.

However, from the diverse literature, it appears that there has been no open research that specifically addresses the mathematical-geometric basis of the 5° + 8° criterion. In fact, this criterion has long been guided by Muhammadiyah and has recently become one of the important references in the discourse of the preparation of the Unified Global Hijri Calendar (KHGT).

The novelty of this research lies in the effort to conduct a purely geometric-mathematical analysis of the 5° + 8° parameter, which has been used more practically without in-depth theoretical studies. In addition, this study will conduct a systematic validation based on global rukyat data, so that it can test whether the criteria are consistent with the reality of observation. Not only that, statistical tests in the form of accuracy, sensitivity, and specificity will also be applied to provide stronger scientific legitimacy. Thus, this research is expected to present a new foothold that is not only academically relevant, but also practically significant in strengthening the KHGT argument

B. Method

This research is a combination research (mixed methods) because it integrates two different but complementary scientific approaches, namely the qualitative-theoretical approach and the quantitative-empirical approach. [9] The selection of this combination method is not just a variation of technique, but a methodological need born from the nature of the problem being studied. The research problem that focuses on the scientific legitimacy of the criteria of a minimum of 5° hilal height and an elongation of at least 8° cannot be answered with a mathematical-geometric approach alone, or with field data analysis alone. Synergy between theory and observational data is needed so that research results are comprehensive, valid, and can be accounted for academically and practically

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C. Results And Discussion

The Bruins criteria are one of the classic criteria in the study of the visibility of the hilal. Although this criterion was developed before the era of observation using Charge-Coupled Device (CCD) cameras, which are digital imaging devices that use silicon semiconductor sensors to convert light (photons) into electrical signals (electrons) [10] and advanced atmospheric modeling [11], it remains an important reference in the discussion of the first crescent moon sighting.

The criteria were developed based on historical moon observation data. The Bruins' focus is on linking certain astronomical parameters to the possibility of hilal visibility with the direct eye. This formula was developed as an interpolation or approximation of Bruin's (1977) original data, which linked the thickness of the *hilal* to the ARCV so that the hilal could be seen with the direct eye.

$$ARCV = 11.5521745317 – 7.944238328 \, .w' + 3.2608487770 \, .(w')^2 – 0.4559413249 \, . \, (w')^3$$

This formula is used to calculate the minimum Arc of Vision (ARCV) based on the width of the hilal (w') in arcminutes. This formula states that the visibility of the new moon is not linear: the thicker the new moon (the greater the w'), the less ARCV needed to see the new moon

Table 1. Bruin Tables of Value for Lunar Crescent Limiting Visibility

Lunar Width (')	Arc of Vision (°)
0,5	8.45
0,7	7.23
1	6.55
2	5.05
3	4.77

Source : Bruin (1977)

$$W = \textit{Phase Fraction} \times \textit{Angular diameter}$$

Example :

- Phase 3.4% (or 0.034)
- Angular diameter: 30 arcminutes

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W = 0.034 x 30 = 1.02' (arc minutes)

For example we want to calculate the minimum ARCV for the hilal with a width w' = 1.0 arcminutes

$$ARCV = 11.5521745317 – 7.944238328 . 1.0 + 3.2608487770 . (1.0)^2 – 0.4559413249 . (1.0)^3$$

$$ARCV = 11.5521745317 – 7.944238328 + 3.2608487770 – 0.4559413249$$

$$ARCV = 6.41$$

So, for a width of 1.0 arcminutes, the Best Time ARCV is around 6.41° so that it can be seen with the naked eye according to Bruin's criteria.

Bruin emphasized that the hilal will not be visible to the naked eye if the width of the hilal is too thin (<15 arcseconds) or if the ARCV is too small (<5°).

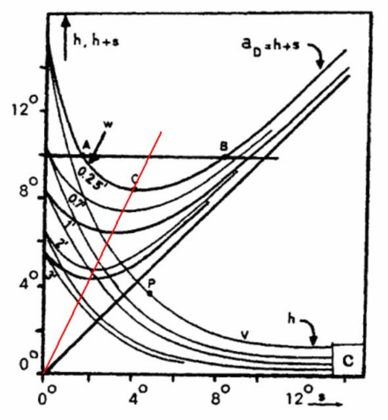

Figure 3. Theoretical Graph of Bruin

Bruin developed a theoretical graph in 1977, which mapped the Moon's altitude and ARCV versus the Sun's depression using various values of crescent width, as shown in Figure 2

For example, with a crescent moon width of 0.25' and an ARCV of 10°, the crescent moon will remain visible as long as the Sun is between 2~8° depression (points A and B).

Meanwhile, the maximum Phase Fraction of the month on the second day is 5% so that the width of the moon is 1.6'. If you look at the Theoretical graph from Bruin, the best time height is 5 degrees, then the maximum ARCV of the moon at the width of the moon is 1.6', which is 5 degrees, if below 5 degrees the moon's width is automatically more than 2',

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then the moon is already very large, no longer included in the moon's visibility in determining the beginning of the month.

So scientifically the height parameter of 5 degrees on the KHGT, was adopted and inspired by the results of Bruin's research in 1977

As for the 8-degree elongation parameter, Danjon states that the first crescent moon is impossible to see with the naked eye if the elongation angle between the moon and the sun (otherwise known as ARCL - Arc of Light) is less than 7 degrees. This theory was put forward by Danjon in 1936.

This is reinforced by the following observation data:

The minimum elongated crescent moon visible to naked eye observation is 7.7 degrees (Pierce #274, at the time it was first seen at 23:55 UT.). for optical assistance is 6.4 degrees (Stamm #797, at the last time seen at 13:09 UT) (Odeh, 2006)

Therefore, the 8-degree parameter is very likely to be inspired by the danjon criterion and is the Best Time combination of the Bruin criterion. This parameter is also still relevant today with the findings or record of the minimum elongation visible to direct eye observation is 7.7 degrees.

Therefore, it is scientifically logical that the combination of the parameters 5° + 8° is applied in the criteria for the formulation of the Unified Global Hijri calendar, because if the parameters are too low it will result in the Eastern end of the world in many cases often not having Ijtimak before dawn, When it happens it will result in a combination of the number of days in the Hijri month that is not ideal between 29 days and 30 days in a year and if the criteria are raised it is feared that the new moon can be seen However, it has not entered the beginning of the new month it is contrary to the calendar requirements.

D. Conclusion

Based on theoretical, mathematical, and observational data, it can be concluded that the parameters of the Unified Global Hijri Calendar (KHGT) criteria in the form of a minimum hilal height of 5° and a minimum elongation of 8° have a strong, rational, and relevant scientific foundation to the development of modern astronomy. The 5° height parameter does not appear arbitrarily, but is closely related to Bruin's 1977 theoretical analysis that mapped the relationship between the Moon's height, ARCV, solar depression,

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and crescent width of the crescent. In the theoretical graph, the best time visibility conditions show that at a certain crescent moon, specifically about 1.6', the most ideal position of the moon is at a height of about 5°. Under these parameters, mathematically the width of the crescent will increase beyond the ideal limit of visibility at the beginning of the month, so that the moon no longer represents the initial phase of the new moon's appearance astronomically.

Meanwhile, the elongation parameter of 8° has a strong correlation with the theory of the visibility limit of the hilal proposed by André Danjon in 1936. Danjon's theory states that the first moon is almost impossible to see with the naked eye when the Moon-Sun elongation is less than 7°. This finding was later strengthened by modern empirical observational data which showed that the minimum elongation of the hilal that was successfully seen with the naked eye was around 7.7°, while with the help of optics it was around 6.4°. Thus, the determination of 8° elongation in the KHGT can be understood as a form of scientific refinement that combines the theoretical limits of Danjon with the optimal visibility approach of the Bruin analysis.

Therefore, the combination of 5° + 8° parameters in the KHGT is a formulation that balances the astronomical, mathematical, and needs of the global calendar system. These parameters are low enough to keep the likelihood of the beginning of the month consistent around the world, but still tight enough that the lunar new moon on which the calendar is based is actually in the visible probability zone. Scientifically, this criterion is also able to maintain the regularity of the Hijri month cycle between 29 and 30 days proportionally, so that it does not cause calendar distortion due to criteria that are too loose or too high. Thus, the KHGT criteria can be seen as the result of a mature scientific synthesis between the theory of hilal visibility, empirical observation data, and the need for the unification of the global Hijri calendar.

References

[1] N. et al Ahmad, “A New Crescent Moon Visibility Criteria using Circular Regression Model : A Case Study of Teluk Kemang , Malaysia,” *Sains Malaysiana*, vol. 49, no. 4, pp. 859–870, 2020.

[2] M. S. Odeh, “New criterion for lunar crescent visibility,” *Exp. Astron.*, vol. 18, no. 1–3, pp. 39–64, 2004, doi: 10.1007/s10686-005-9002-5.

[3] M. Hidayat, "Aplikasi Kriteria Kalender Islam Global Muktamar Turki 2016 dan Rekomendasi Jakarta 2017," *Al-Marshad J. Astron. Islam dan Ilmu-Ilmu Berkaitan*, vol. 4, no. 1, p. 73, 2018.

[4] A. J. Rakhmadi and M. Hidayat, "The Issues and Prospects of the Global Islamic Calendar," vol. 477, no. Iccd, pp. 109–112, 2020, doi: 10.2991/assehr.k.201017.025.

[5] Muhammad Hidayat et al, "PROBLEM AND SOLUTIONS FOR ACCEPTING A UNIFIED GLOBAL ISLAMIC CALENDAR," *J. Law Gov.* , vol. 7, no. 1, pp. 63–72, 2024.

[6] M. Syazwan Faid, M. S. A. Mohd Nawawi, M. H. M. Saadon, and N. Ahmad, "A Review on Modern Lunar Crescent Visibility Criterion," *Malaysian J. Sci.*, vol. 44, no. 1, pp. 101–120, Mar. 2025, doi: 10.22452/mjs.vol44no1.12.

[7] Danjon, "Le crissant lunaire [The Lunar Crescent]. ," *L'Astronomie Bull. La Soc. Astron. Fr.* , pp. 57–65, 1936.

[8] M. Ilyas, "Lunar Crescent Visibility Criterion and Islamic Calendar," *Q. J. R. Astron. Soc.* , vol. 35, no. 4, pp. 425–461, 1994.

[9] S. Dawadi, S. Shrestha, and R. A. Giri, "Mixed-Methods Research: A Discussion on its Types, Challenges, and Criticisms," *J. Pract. Stud. Educ.*, vol. 2, no. 2, pp. 25–36, Feb. 2021, doi: 10.46809/jpse.v2i2.20.

[10] R. Chiranjeevi *et al.*, "Analyzing the Electronics of Image Sensors and Their Functionality to Develop Low Light-Emitting Source Image," *Appl. Syst. Innov.*, vol. 5, no. 5, p. 100, Oct. 2022, doi: 10.3390/asi5050100.

[11] B. Munir, "Faktor Atmosfer Dalam Visibilitas Hilal Menurut Badan Meteorologi Klimatologi," *Literasi Unggulan Ilm. Multidisipliner*, vol. Vol. 2, no. No. 4, p. h. 684, 2024.