

Estimation of Ductility Demand in RC Buildings Induced by Repeated Earthquakes

Ade Faisal^{1*}, Taksiah A. Majid²

¹Program Studi Teknik Sipil, Universitas Muhammadiyah Sumatera Utara (UMSU), Medan, Indonesia

²Disaster Research Nexus, Universiti Sains Malaysia, P. Pinang, Malaysia

*adefaisal@yahoo.com

Abstract

Current design practice considers the use of single event of earthquake. In fact, seismic hazard is a multi events earthquake or the so-called repeated earthquakes. Consequently, the structural behavior under repeated earthquakes is not clearly understood. Therefore, the present study focused on the estimation of ductility demands in reinforced concrete (RC) buildings affected by repeated near-field earthquake having forward directivity effect (FDE). A comprehensive assessment was conducted using generic frames with 4 types of fundamental period. A model having behavior factor (or force reduction factor) of 1.5, 2, 4, and 6.0, and plastic hinge at member ends with 3 types of plastic rotation capacity was assumed. The buildings were assumed to be situated on a stiff soil in the high seismic zone in Europe. This study shows that, on average, the amplification ratio of roof ductility demand due to repeated earthquakes reached to 1.5 and 1.7 for double and triple events of repeated earthquakes, respectively. The present study has also established the empirical relationships of ductility demands of RC building with the fundamental period, behavior factor, ratio of global post-yield stiffness to elastic stiffness, and ratio of story ductility to global ductility capacities to predict the amplification ratios of ductility demand due to repeated earthquakes.

Keywords: Ductility demand, behavior factor, repeated earthquake, reinforced concrete

1.0 Introduction

In design practice, the building is arranged to perform well in its inelastic state under such level of earthquake. To sustain in the inelastic state, the elastic design spectrum acceleration for a building is modified using a behavior factor. The design base shear and story shear in the structure are reduced by this behavior factor to account for the inelastic force. Under this condition, the design displacements are assumed to be essentially unchanged by the inelastic force (Iwan et al., 2000). This design process is a well-known seismic design concept, known as equal displacement principle. Having this concept, the inelastic force is not necessarily defined using nonlinear analysis. It can be just simply calculated by using linear elastic analysis. It should be underlined that the aforementioned seismic design process is aimed to produce a building that able to sustain damage from a single event of earthquake. In fact, the earthquake hazard usually does not occur as a single event, as mostly assumed in the seismic design, but as a series of shocks. The strong earthquakes have more and larger aftershocks, sometimes foreshocks, and the sequences can last for years or even longer. The aftershocks are usually unpredictable and can be of a large magnitude, which could collapse the buildings that are damaged from the mainshock (Elnashai et al., 1998).

The repetition of medium-strong earthquake ground motions after intervals of time is characterized as the repeated earthquake. The interval of time could be short or long. Figure 1 demonstrates the combination of foreshock, mainshock, and aftershock in repeated earthquakes, which is based on the 1984 Umbria earthquake motion recorded from Norcera Umbra station, Italy. The repeated earthquakes are a series of foreshock, mainshock and aftershock within a range of time. Faisal (2012) found that the repeated earthquakes can also be a combination of near-field and far-field earthquakes containing ground motions with forward directivity (pulse-effect) and backward directivity (no pulse effect) effects. The study concluded that the character of repeated earthquakes is indicated by the series of earthquake shock (regardless the foreshock, mainshock, and aftershock) sourced from the various type of earthquake with medium-strong intensity level within any range of time, but not more than the building's design life span (i.e., 50 years).

Despite the fact that the repeated earthquakes hazard was clearly threatening, the effect of repeated earthquakes on the structures has not attracted much attention (Hatzigeorgiou and Beskos, 2009). The current literature survey has found that few studies have examined the repeated earthquakes effect on the buildings and no related clauses are found in the seismic codes.

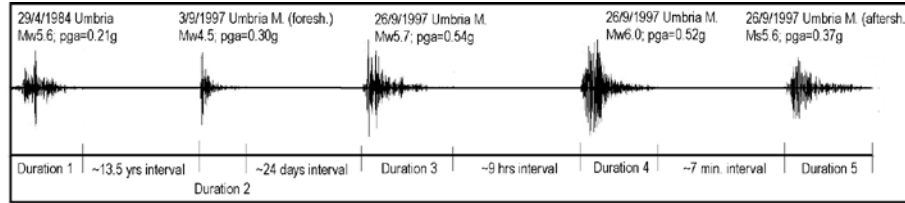


Figure 1: A repeated earthquake ground motions recorded from Norcera Umbra station, Italy

Moreover, the recent studies published have not addressed the needs in engineering design properly. For instance, some studies focused on single-degree-of-freedom system (SDOF) having bilinear elasto-plastic hysteresis with no stiffness and strength degradation (Hatzigeorgiou and Beskos, 2009; Hatzigeorgiou, 2010). Recently, Hatzigeorgiou and Liolios (2010) have tried to address the lack in MDOF system issue. They have used 2 RC frame models to represent 2 types of fundamental period of vibration of structures with unvaried behavior factor. The number of model and behavior factor used in the study was few in order to represent all types of regular RC frame building. These drawbacks would lead to underestimation of the result of designed drift of buildings, which could endanger the occupant when repeated earthquakes occurred. Therefore, the present study aims to determine the effect of near-field repeated earthquakes on the roof ductility of inelastic RC buildings and to develop the empirical relationship of roof ductility demands with the behavior factor, fundamental period, ratio of post-elastic stiffness, and ratio of roof and story ductility capacity for the inelastic RC buildings due to the repeated near-field earthquakes as a tool in seismic evaluation.

2.0 Inelastic Structure Models

2.1 Generic Frames

The moment resisting frame system in form of generic frame model with 3D multi-storey single-bay system developed by Faisal (2012) is used in the present work. It is developed based on concept in Ruiz-Garcia and Miranda (2005), which was previously proposed using 2D generic frames. This type of generic frame has also been used in Ibarra et al. (2005) and extended to multi-bays in Zareian and Krawinkler (2009). The regular geometric system using 3, 6, 12, and 18-story single-bay frame models (Figure 2a) are selected, which is comply with Eurocode 8 and ASCE 7-05 as a regular system in horizontal and elevation. Considered fundamental periods of the models are $T_1 = 0.45, 0.75, 1.26,$ and 1.71 seconds. The plan shape of floor and roof is squared plan size of 7.2×7.2 m with column height is equal to 3.6 m for all stories uniformly. Columns and beams at each story have the same stiffness in order to reduce the uncertainty in modelling. The member inelasticity is modeled by flexural plastic hinges located at the member ends (full-hinge mechanism) and a 10% of member length from beam-column joint is assumed for plastic hinge length.

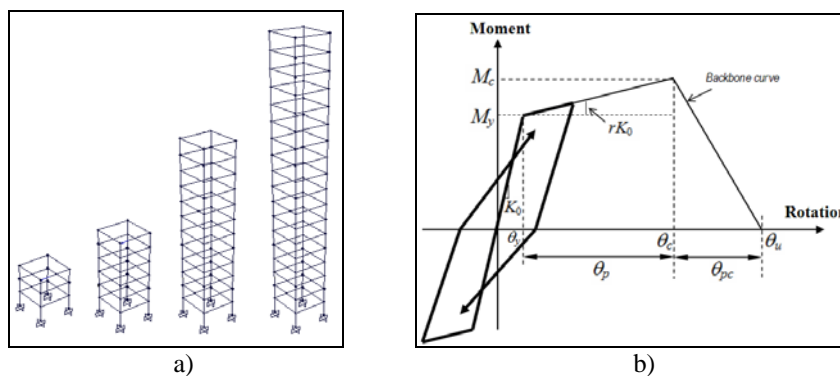


Figure 2: a) Models of 3D generic 3, 6, 12, and 18-story single-bay frames, and b) Modified-Takeda hysteresis and backbone curve

2.2 Stiffness and Strength Distributions

The stiffness distribution used for the models is classified as regular stiffness according to ASCE 7-05, which means the difference between the stiffness of adjacent stories is $<60\%$ of the story above or $<70\%$ of the average stiffness of the three stories above. This condition complies with Eurocode 8 as regular stiffness as well. The models are designed to exhibit a first-mode elastic deflected shape in order to be more realistic. Therefore,

we have used the ratio of the stiffness of all beams at the mid-height story of the frame to the sum of the stiffness of all columns at the same story based on empirical relationship of Miranda and Taghavi (2005). The stiffness distribution is tuned so that the variation of lateral stiffness along the height of the structure follows parabolic variations (Miranda and Reyes, 2002).

Four type of ductility-related behavior factor, q , namely 1.5, 2, 4, and 6, are considered. Definition of the flexural plastic hinge is in line with the selected q using linear elastic analysis. This definition is also employed, among others, by Medina and Krawinkler (2003), Ruiz-Garcia and Miranda (2005), Zareian and Krawinkler (2009). To adopt strong column - weak beam mechanism in earthquake resistant system, 1.3 strength ratios of column and beam is initially considered ($\Sigma M_b \leq 1.3 \Sigma M_{col}$), as suggested by Eurocode 8, in obtaining the height-wise strength distribution. The overstrength is also added uniformly by a factor 1.3 of the available strength to represent the overstrength regulated in Eurocode 8 for real multi-story multi-bay structures. The base shear force is defined from ordinate design spectrum at period T_1 of Type 1 spectrum of Eurocode 8 for condition of Soil B with peak ground acceleration (PGA), $a_g = 0.36$ g. The a_g is based on 475-years return period of earthquake that reflecting the condition of Seismic Zone III in Greece. Greece represents the highest seismic region in Europe, along with Turkey and Italy, whereas Zone III is the highest seismic zone in Greece.

2.3 Plastic Hinge and Backbone Curve

The plastic hinge is represented by moment-rotation relationship modelled using lumped plasticity model. To simulate the cyclic behavior of RC members in plastic hinge under load reversals, Modified-Takeda hysteresis rule is employed (Figure 2b) with the unloading and reloading parameters ($\alpha = 0.3$ and $\beta = 0.6$) for beam and column member are identical, as suggested by Fardis (2007). The backbone curve proposed by Zareian and Krawinkler (2009) is used, as shown in Figure 2b. To represent the capacity of general RC structures, this study employs the moment-rotation capacities of RC beam-column member within the range of capacity suggested by Haselton et al. (2007). To reflect low, medium, and high rotation capacities of RC member, this study has used plastic rotation capacity, θ_p , namely 0.02, 0.04, and 0.06, as proposed by Zareian and Krawinkler (2009). The post capping rotation, θ_{pc} , is assumed equal to 0.06 based on the average value of θ_{pc} used in Zareian and Krawinkler's study, whereas the ratio of M_c/M_y is assumed to be 1.13, as suggested by FEMA-P695.

2.4 Global Inelastic Characteristics

Based on nonlinear static analysis, the parameters of ratio of global post-yield stiffness to elastic stiffness, r_K , and the ratio of ductility capacity, \mathcal{G}_c , are introduced in order to characterize the global stiffness of the models. After regressing r_K of 60 models having varies T_1 , q , and θ_p , the following expression is used to characterize the global stiffness of the considered models in the present work.

$$r_K = 10^{-0.132T_1^3 - 0.901\text{Log}(q) - 11.252\theta_p - 1.290} \quad (1)$$

The coefficient of determination, denoted by R^2 , and standard deviation of error in log-normal distribution, denoted as σ_{err} , for empirical relation in Eq. (1) is equal to 0.972 and 0.072, respectively. The relationship explains that θ_p governs r_K compares with T_1 and q . The trend also clearly shows that as θ_p increases r_K decreases, a trend similar to that in the Zareian and Krawinkler's (2009) study. Only r_K with the range of 0% - 3% is evaluated in the current work. Moreover, the ratio of ductility capacity of the models represented by the ratio of story to global ductility capacity is introduced to relate the maximum global ductility (based on roof displacement) with the maximum story ductility (based on interstory drift) capacity. The following empirical relationship of \mathcal{G}_c reflects the ratio of ductility capacity of the 60 models, based on regression analysis ($R^2 = 0.983$; $\sigma_{err} = 0.028$).

$$\mathcal{G}_c = 10^{0.513T_1^3 - 0.971T_1^2 - 0.406T_1^{-1} - 0.046\text{Log}(r_K) + 1.037} \quad (2)$$

The aforesaid relationship excludes q and θ_p since both have negligible effects on \mathcal{G}_c . It also clearly indicates that as T_1 increases, \mathcal{G}_c increases. The models evaluated in the present work were having the ratio of ductility capacity of $1.2 \leq \mathcal{G}_c \leq 4.9$.

3.0 Input Motion

3.1 Selection and Scaling Ground Motion Record

The method of selection of ground motion record follows the method suggested by Bommer and Acevedo (2004). The ground motion records containing the large pulse in FDE, based on the list given by Baker

(2007) are selected from the Pacific Earthquake Engineering Research (PEER) Database. Number of the selected strong motion records for each type of ground motion is 20 records.

Based on suggestion by Baker (2007), a FDE record comprises of large pulse signal and regular signal (without large pulse). The study explains that the regular motion component (so-called residual FDE) behaves in a similar manner as that of FFE motions or NFGM having backward directivity. Furthermore, a real orthogonal component of FDE normally contains only one large pulse component because the velocity amplitude of fault-parallel component is in average 65% lower than the fault-normal component. The pulse period of velocity has also showed similar trend (Rodriguez-Marek and Bray, 2006). Therefore, to incorporate the FDE having a single component of large pulse in its horizontal components, 20 synthetic residual FDEs of Baker (2007), denoted as RFDE, are also used. The motion is coupled with the FDE having large pulse in bi-directional excitation to represent other horizontal component having no large pulse motion. Such consideration is adopted based on the reason that the RFDE motion could reflect the same seismic source regimes with its FDE. Moreover, it is also due to the reason that both motions would have the same oscillation period and duration (Baker, 2007).

The spectrum acceleration at fundamental period of structure, denoted as $S_d(T_1)$, is utilized as the intensity measure of ground motion in nonlinear dynamic analysis. The scaling process focuses on this intensity measure following the method proposed by Shome et al. (1998). In this method, all ground motions is scaled to the same pseudo-spectrum acceleration provided by Eurocode 8 at the first mode period of structure, T_1 . It is employed since it involves simple process and provides a relative accuracy (Giovenale et al., 2004).

The minor component of orthogonally paired ground motions is proportionally scaled relative to its corresponding major component. In this study, the minor component is scaled following the ratio of major-minor components of its original record. In this way, the frequency content of both components, as well as the intensity of one relative to the other, would be maintained.

3.3 Assembling Ground Motion to Represent Repeated Earthquakes

In this study, the repeated earthquakes present in the form of a combination of ground motion with single, double and triple events. The method of assembly is taken from Hatzigeorgiou and Beskos (2009). In this method, the amplitude ratio of assembled ground motion is scaled based on the ratio of peak ground acceleration (PGA), which is governed by the magnitude within a consecutive earthquakes sourced from the same seismic region and recorded at the same site. The ratio of PGA is derived using the ratio of empirical attenuation functions, which varies in magnitude. Each of FDE and RFDE events is applied for GM Case 1 (single earthquake event). For GM Case 2 and 3 of FDE are assembled by randomly adding the RFDE event into the initial FDE of GM Case 1. The ratio of PGA for repeated earthquakes is as follows (Figure 3):

Single event, GM Case 1: (1.000, 0.000, 0.000)

Double events, GM Case 2: (1.000, 1.000, 0.000)

Triple events, GM Case 3: (0.853, 1.000, 0.853)

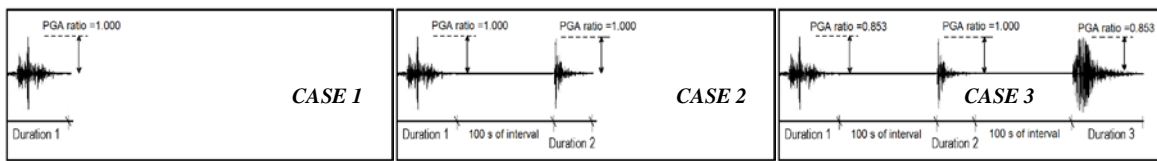


Figure 3: Illustration of assembling the ground motion to generate synthetic repeated earthquakes

4.0 Result and Discussion

The result is found based on nonlinear static analysis and nonlinear time history analysis using Ruaumoko program. The influence of repeated earthquakes on the roof ductility demand, μ_A , of models having $T_1 = 0.45, 0.75, 1.26$, and 1.71 s and mean θ_p are demonstrated in Figure 4. It is clearly apparent that for all models with lower behavior factor (i.e., $q < 2$), the effect of repeated FDE is insignificant and can be negligible. For the case of 3-story model, the effect of repeated FFE on μ_A is not apparent. Figures 4 also explain that μ_A is dominantly affected by GM Case 3. However, the gap of roof ductility demands under repeated FDE of GM Case 2 and 3 on the models having $T_1 \geq 0.75$ s and $q \leq 4$ is relatively small and hence can be neglected. The effect of repeated earthquakes in the form of roof ductility demand due to GM Case 2 or 3 is normalized by GM Case 1, denoted as amplification ratio due to repeated earthquakes, is summarized in Table 1.

The trend of $A_{r-\mu_A}$ on all models excited by GM Case 2 and 3 is arbitrary due to the presence of q . It is very common to have the q trend for models with various T_1 under the same μ_A is arbitrary, as explained in

Kappos (1999) and Hatzigeorgiou (2010). The pattern of q trend is approximately similar when μ_Δ of model with the various T_1 is increased. the models with $q \geq 4$ excited by GM Case 2 and 3 show that $A_{r-\mu_\Delta}$ is reduced by the increase of number of story of models (or T_1). It is said so because as the flexibility of building with DCH increases, the effect of repeated earthquakes on μ_Δ decreases.

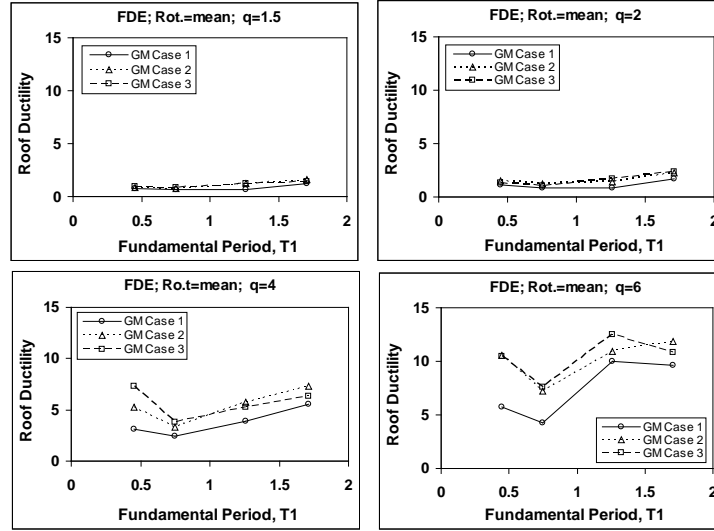


Figure 4: Roof ductility demand as a function of T_1 and mean θ_p of models with $q = 1.5$ to 6

Table 1: Amplification ratio of roof ductility demand, $A_{r-\mu_\Delta}$, on the models with $q \geq 1.5$ and mean θ_p

Models	Repeated GM case	Behavior factor			
		$q = 1.5$	$q = 2$	$q = 4$	$q = 6$
3-story model ($T_1 = 0.45$ s)	GM Case 2	1.2	1.3	1.7	1.8
	GM Case 3	1.2	1.1	2.4	1.8
6-story model ($T_1 = 0.75$ s)	GM Case 2	1.3	1.5	1.4	1.7
	GM Case 3	1.3	1.3	1.6	1.8
12-story model ($T_1 = 1.26$ s)	GM Case 2	1.9	1.7	1.5	1.1
	GM Case 3	2.0	2.1	1.3	1.3
18-story model ($T_1 = 1.71$ s)	GM Case 2	1.3	1.4	1.3	1.2
	GM Case 3	1.1	1.5	1.2	1.2

For lower behavior factor such as $q < 4$, the repeated earthquakes GM Case 2 and 3 of FDE increase $A_{r-\mu_\Delta}$ on 3-story up to 12-story models. Thus, $A_{r-\mu_\Delta}$ is decreased significantly on the 18-story models. Note that the aforementioned conditions are strictly applied to the case of repeated FDE containing a ground motion with large pulse (in main-shock) in its seismic sequences. It is obvious that this arbitrary trend of $A_{r-\mu_\Delta}$ is governed by the dissimilarity trend of μ_Δ quantity of the models excited by GM Case 2 and 3 in comparison with GM Case 1. The dissimilarity trend of μ_Δ quantity is mainly contributed by T_1 and q . It is due to the reason that the trend of μ_Δ is random for various T_1 under the same q , as explained in Lam et al. (1996). Moreover, there is also arbitrary trend of q for various T_1 under the same μ_Δ , as explained in Kappos (1999) and Hatzigeorgiou (2010). $A_{r-\mu_\Delta}$ of models excited by repeated FDE is found to be equal to, in average, 1.5.

This study proposed empirical relationships to estimate mean roof ductility demand, μ_Δ , for models excited by GM Case 1, 2, and 3 of FDE, respectively as follows:

$$\begin{aligned} \ln(\mu_{\Delta, D1}) = & -0.199\ln(T_1) + 0.931\ln(q) + 0.185(T_1 \cdot q) \\ & -13.133r_K - 0.844 \end{aligned} \quad (3)$$

$$\begin{aligned} \ln(\mu_{\Delta,D2}) = & 2.290\ln(T_1) + 1.520\ln(q) + 1.509(T_1^{-1}) \\ & - 4.343r_K - 0.078g_c - 1.960 \end{aligned} \quad (4)$$

$$\begin{aligned} \ln(\mu_{\Delta,D3}) = & 2.551\ln(T_1) + 1.522\ln(q) + 1.671(T_1^{-1}) \\ & - 4.527r_K - 0.096g_c - 2.053 \end{aligned} \quad (5)$$

where R^2 for these relationships are equal to 0.963, 0.981, and 0.978, respectively. The standard deviation of error, σ_{est} , for Eq. 3 to 5 are equal to 0.212, 0.149, and 0.161, respectively.

In most practical engineering design and evaluation, it is more convenient to have a tool in the form of ratio in assessing the effect of two factors on the seismic performance of structures. Therefore, the influence of repeated earthquakes on μ_{Δ} of models can be easily understood in form of the amplification ratio, A_r , of $\mu_{\Delta,jn}$ due to GM Case 2 or 3 (or both) to the ductility demand due to GM Case 1 [e.g., $A_{r-\mu_{\Delta}(F2/1)} = \ln(\mu_{\Delta,F2}) / \ln(\mu_{\Delta,F1})$; or $A_{r-\mu_{\Delta}(D3/1)} = \ln(\mu_{\Delta,D3}) / \ln(\mu_{\Delta,D1})$, respectively]. Using this ratio, the amplification of drift in models excited by repeated earthquakes can be simply estimated after conducting a regular procedure in seismic evaluation by multiplying the drift result to this ratio.

5.0 Conclusions

The present work concludes that the repeated earthquakes affect the roof ductility demand significantly. On average, the amplification ratio of roof ductility demand is found to be equal to 1.5 and 1.7 for the RC buildings excited by double and triple near-field earthquakes events. The amplification ratio of roof ductility demand is not mainly governed by the fundamental period. The short-period frame models with medium ductility class might have larger amplification ratio of roof ductility demand than the long-period frame models. In general, the trend of amplification ratio of roof ductility demand is arbitrary due to the presence of behavior factor. The present study proposed the empirical relationships of roof ductility demands of RC buildings with the fundamental period, behavior factor, ratio of global post-yield stiffness to elastic stiffness, and ratio of story ductility to global ductility capacities to predict the amplification ratios of roof ductility demand due to repeated near field earthquakes having forward directivity effect.

References

- ASCE. (2005). *Minimum design loads for buildings and other structures*, ASCE/SEI 7-05. Reston, VA.: ASCE.
- Baker, J. W. (2007). Quantitative classification of near-fault ground motions using wavelet analysis. *Bulletin of the Seismological Society of America*, 97(5), 1486-1501.
- Bommer, J. J., and Acevedo, A. B. (2004). The use of real earthquake accelerograms as input to dynamic analysis. *Journal of Earthquake Engineering*, 8(1 SI), 43-91.
- Bray, J. D., and Rodriguez-Marek, A. (2004). Characterization of forward-directivity ground motions in the near-fault region. *Soil dynamics and earthquake engineering*, 24(11), 815-828.
- CEN. (2004). *Eurocode 8: Design of structures for earthquake resistance. Part 1: General rules, seismic actions and rules for buildings* (Vol. 1). Brussels: European Committee for Standardization.
- Elnashai, A. S., Bommer, J. J., and Martinez-Pereira, A. (1998). Engineering implications of strong-motion records from recent earthquakes. Paper presented at the *11th European Conference on Earthquake Engineering*.
- Faisal, A. (2012). *Influence of repeated earthquakes on the ductility demand of inelastic RC buildings*, PhD Theses, Universiti Sains Malaysia.
- Fardis, M. N. (2009). *Seismic design, assessment and retrofitting of concrete buildings: based on EN-Eurocode 8* (Vol. 8). New York: Springer.
- FEMA. (2009). *Quantification of building seismic performance factors*, FEMA P695. Washington, DC: Federal Emergency Management Agency.
- Giovenale, P., Cornell, C. A., and Esteva, L. (2004). Comparing the adequacy of alternative ground motion intensity measures for the estimation of structural responses. *Earthquake Engineering and Structural Dynamics*, 33(8), 951-979.
- Haselton, C. B., Liel, A. B., Lange, S. T., and Deierlein, G. G. (2007). *Beam-column element model calibrated for predicting flexural response leading to global collapse of RC frame buildings*. Report No. 2007/03, Pacific Earthquake Engineering Research Center, University of California at Berkeley.
- Hatzigeorgiou, G. D. (2010). Ductility demand spectra for multiple near- and far-fault earthquakes. *Soil Dynamics and Earthquake Engineering*, 30(4), 170-183.
- Hatzigeorgiou, G. D., and Beskos, D. E. (2009). Inelastic displacement ratios for SDOF structures subjected to repeated earthquakes. *Engineering Structures*, 31(11), 2744-2755.

- Hatzigeorgiou, G. D., and Liolios, A. A. (2010). Nonlinear behaviour of RC frames under repeated strong ground motions. *Soil Dynamics and Earthquake Engineering*, 30(10), 1010-1025.
- Ibarra, L. F., Medina, R. A., and Krawinkler, H. (2005). Hysteretic models that incorporate strength and stiffness deterioration. *Earthquake Engineering and Structural Dynamics*, 34(12), 1489-1511.
- Iwan, W. D., Huang, C. T., and Guyader, A. C. (2000). Important features of the response of inelastic structures to near-field ground motion. Paper presented at the *12th World Conference on Earthquake Engineering*.
- Kappos, A. J. (1999). Evaluation of behaviour factors on the basis of ductility and overstrength studies. *Engineering Structures*, 21(9), 823-835.
- Lam, N., Wilson, J., and Hutchinson, G. (1996). Building ductility demand: interplate versus intraplate earthquakes. *Earthquake Engineering and Structural Dynamics*, 25(9), 965-985.
- Miranda, E., and Reyes, C. J. (2002). Approximate lateral drift demands in multistory buildings with nonuniform stiffness. *Journal of Structural Engineering*, 128(7), 840-849.
- Miranda, E., and Taghavi, S. (2005). Approximate floor acceleration demands in multistory buildings. I: Formulation. *Journal of Structural Engineering*, 131(2), 203-211.
- Medina, R. A., and Krawinkler, H. (2005). Evaluation of drift demands for the seismic performance assessment of frames. *Journal of Structural Engineering*, 131(7), 1003-1013.
- Rodriguez-Marek, A., and Bray, J. D. (2006). Seismic site response for near-fault forward directivity ground motions. *Journal of Geotechnical and Geoenvironmental Engineering*, 132(12), 1611-1620.
- Ruiz-Garcia, J., and Miranda, E. (2005). *Performance-based assessment of existing structures accounting for residual displacements*. Report No. 153, John A. Blume Earthquake Engineering Center, Stanford University.
- Shome, N., Cornell, C. A., Bazzurro, P., and Carballo, J. E. (1998). Earthquakes, records, and nonlinear responses. *Earthquake Spectra*, 14(3), 469-500.
- Zareian, F., and Krawinkler, H. (2009). *Simplified performance-based earthquake engineering*. Stanford: Report No. 169, John A. Blume Earthquake Engineering Center, Stanford University.