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On The Rainbow Connection Of Middle Graph Of Firecracker Graphs $(F_{n,4})$

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ABSTRACT

Coloring in graph theory includes various approaches, one of which is rainbow coloring which is closely related to the concept of rainbow connected numbers which refers to the least number of colors needed to color the edges in a graph so that every two vertices connected in a rainbow path have the same color and is denoted by $rc(G)$. Rainbow coloring can be studied in several forms of graph development, one of which is the middle graph. All types of graphs, both simple and complex, can be represented as a middle graph. A middle graph is a graph whose vertices are obtained from the vertices and edges of graph G and is denoted by $V\ (M(G)) = V\ (G) \cup (G)$. Two points in a middle graph are considered adjacent if and only if they are adjacent edges in G or one of the points is adjacent to an edge of G. In this research, we discuss the number $rc(G)$ on the middle graph of firecracker graph $(F_{n,4})$ with $n \geq 2$. Based on the research results, we obtain the rainbow connected number theorem on the middle graph of firecrackers graph $rc(M(F_{n,4})) = 3n + 2$ for $n \geq 2$.

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INTRODUCTION

Humans always face problems in their daily lives and look for the best solutions to overcome them. This indirectly encourages the rapid development of science. Therefore, mathematics has an important role in the advancement of science (Lihawa et al., 2022). Therefore, education is very important for one generation to the next (Akrim, 2020; Hidayat, 2024; Simbolon, 2024; Sri 2024). Graph theory is one part of the branch of mathematics that is useful for expressing or describing a problem so that it is easier to understand and solve (Afifuddin & Budayasa, 2022). A Swiss mathematician named Leonhard Euler discovered graph theory for the first time in 1736. Euler successfully solved the Konigsberg Bridge problem in Russia by representing the problem in the form of a graph (Fransiskus Fran Afriantini, 2019).

The field of graph theory contains a variety of interesting topics, including graph labeling and coloring. A labeling on a graph is a function that maps a vertex or edge, or both, to a certain state (Gallian, 2022). There are three types of graph labeling: point labeling, edge labeling, and total labeling involving both points and edges (Imelda & Martini, 2022). Graph coloring is the process of assigning colors to the elements of a graph G so that each neighboring element has an unequal color. This is very important in determining the minimum number of colors needed to color a graph (Fransiskus Fran Afriantini, 2019). The concept of coloring in graph theory encompasses various approaches, one of which is rainbow coloring which discusses the connected number of rainbows, which refers to the least number of colors needed to color the edges in a graph so that every two connected nodes in the rainbow path have the same color (Chartrand et al., 2008).

Until now, research on rainbow connected numbers has been conducted by several researchers such as those conducted by (Y. J. H. Fransiskus Fran, 2019) and (Lakisa et al., 2022) which discuss the determination of rainbow connected numbers on several types of graphs. Furthermore, research conducted by (Ismail et al., 2022) and (Lihawa et al., 2022) discusses the rainbow connected number of the result of the operation of two graphs. (Kumala, 2019) examines the determination of the rainbow connected number of two new graph classes. Then research by (Yuniarti et al., 2023) examines the rainbow connected number on double quadrilateral snake graphs and alternative double quadrilateral snake graphs.

Topic of rainbow coloring can be studied in several forms of graph development, one of which is the middle graph. A middle graph is a graph whose nodes are obtained from the nodes and edges of a graph G and is denoted by $V(M(G)) = V(G) \cup E(G)$. Two nodes in a middle graph are considered to be neighbors if and only if they are neighbors of an edge in G or one of them is neighbors of an edge in G (Vaidya, Samir K and Bantva, 2010). In the research conducted by (Kim, 2022), the dominated chromatic numbers of all middle graphs were determined. In (Rahmawati et al., 2020) determined the exact value of the total rainbow connected number on the line, square, and middle graphs of the centipede graph (C_n). The research by (B. G. V. Y. Fransiskus Fran, 2020) studied rainbow dot coloring for quadratic graph of firecrackers graph $(F_{n,3}^2)$ and line graph of firecrackers graph $(L(F_{n,3}))$ with $n \geq 2$.

Based on previous studies, research has been conducted on various types of graphs to analyze the structure of the middle graph in a graph. Therefore, in this research, the author is interested in studying the topic of rainbow connected numbers related to the middle graph of firecrackers graph $(F_{n,4})$ which is a combination of path graph and star graph.

As for the theorems and propositions used in this study:
Theorem 1. (Chartrand et al., 2008) Suppose G is an arbitrary graph and $diam(G)$ is the diameter of the graph G, valid:

$$rc(G) \geq diam(G)$$

Proposition 1. (Chartrand et al., 2008) Suppose G is a nontrivial connected graph of size m, then $rc(G) = m$ if and only if G is a tree.

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RESEARCH METHOD

The method used in this research is a literature study (library research), namely by looking for all information from several sources such as scientific articles, journals, books, textbooks and other references related to the research topic, namely rainbow connected numbers and middle graphs. The goal is to obtain information and methods used in the discussion of related issues.
The stages of research carried out in this study:

1) Draw the middle graph $M(G)$ of the firecracker graph $\left(F_{n\ 4}\right)$.
2) Determine and derive the rainbow connected number pattern $rc(G)$ obtained from the previous stage drawing.
3) Proving the rainbow connected number theorem $rc(G)$ on the graph obtained previously.
4) Formulate conclusions based on the findings

RESULTS AND DISCUSSION

Middle Graph of a Firecracker Graph $M(F_{n,4})$

Definition 1. Suppose n is an integer with $n \geq 2$. $M(F_{n,4})$ is the graph G formed through the middle graph formation of the firecracker graph $(F_{n,4})$. The vertex and edge sets of the graph G are defined as follows:

$$V\ (G) = \{u_i\ |i \in [1,n]\} \cup \{u'_i\ |i \in [1,n-1]\} \cup \{v_i\ |i \in [1,n]\} \cup \{v'_i\ |i \in [1,n]\} \cup \{v_{i,j}\ |i \in [1,n], j \in [1,2]\} \cup \{v'_{i,j}\ |i \in [1,n], j \in [1,2]\}$$

$$E(G) = \{u_iu'_i\ |i \in [1,n-1]\} \cup \{u_{i+1}u'_i\ |i \in [1,n-1]\} \cup \{u'_iu'_{i+1}\ |i \in [1,n-1]\} \cup \{u_iv'_i\ |i \in [1,n]\} \cup \{u'_iv'_i\ |i \in [1,n-1]\} \cup \{u'_iv'_{i+1}\ |i \in [1,n-1]\} \cup \{v_iv'_i|i \in [1,n]\} \cup \left\{v_iv'_{i,j}\middle|i \in [1,n], j \in [1,2]\right\} \cup \left\{v'_iv'_{i,j}\middle|i \in [1,n], j \in [1,2]\right\} \cup \left\{v_{i,j}v'_{i,j}\middle|i \in [1,n], j = 1\right\} \cup \left\{v_{i,j}v'_{i,j}\middle|i \in [1,n], j = 2\right\} \cup \left\{v'_{i,1}v'_{i,2}\middle|i \in [1,n]\right\}$$

Rainbow Connection $M(F_{n,4})$

Theorem 2. Suppose n is an integer with $n\ \geq\ 2$ and $G \cong M(F_{n,4})$, then :

$$rc(G) = 3n + 2$$

Proof. Based on the theorem by Chartrand et al. (2008), it is known that $rc(G)\ \geq\ diam(G)$. If $rc(G)\ =\ diam(G)$ then it is sufficient to show the existence of a rainbow trajectory by coloring $c\ :\ E(G)\ \rightarrow\ \{1, 2, \ldots, rc(G)\}$. Whereas if $rc(G)\ >\ diam(G)$, then the proof needs to be done by contradiction. Therefore, the proof of this Theorem 2 is done by contradiction because $rc(G)\ >\ diam(G)$.

Based on the graph in figure 1 there are pendants in an edge $v_{i,j}\ v'_{i,j}$ and a path $v'_{i,j} - v'_{n,j}$, so to make the graph rainbow connected with the minimum color, all edges $v_{i,j}\ v'_{i,j}$ and a

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path $v'_{i,j} - v'_{n,j}$ must be assigned different colors by $3n + 2$. Suppose $rc(M(F_{n,4})) = 3n + 2$ for $n \geq 2$, it will then be shown by contradiction that $rc(M(F_{n,4})) \neq 3n + 1$, with respect to pendant edges of $2n$. Suppose $rc(M(F_{n,4})) = 3n + 1$, then there exists c colors which is a rainbow coloring $c : E(G) \rightarrow \{1,2,\ldots,3n + 1\}$, as shown in figure 1 :

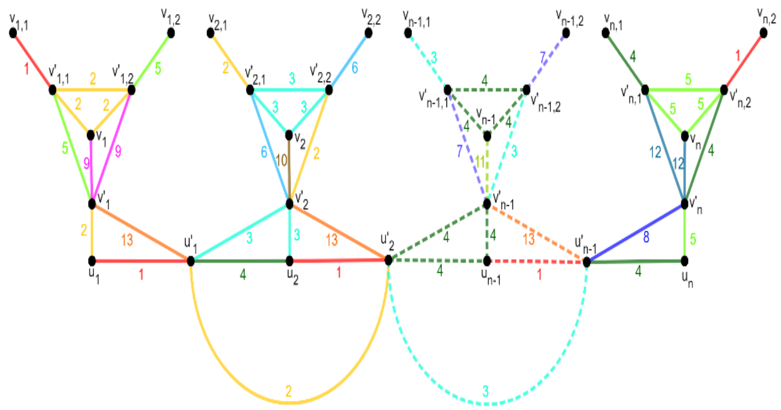

Figure 1. Rainbow Coloring $M(F_{n,4})$ with 13 Colors

Asume a path $v'_{i,j} - v'_{n,j}$ which is a rainbow path and is assigned $n + 2$ different colors. Thus for every pendant in the graph $M(F_{n,4})$ can only be colored with $2n - 1$ colors. While the pendant in graph $M(F_{n,4})$ are as many as $2n$, consequently there is a pendant edge $v_{i,j}\ v'_{i,j}$ which has the same color as the edge $v_{i,j}v'_{i,j}$ which makes the path $v_{i,j} - v_{n,j}$ not rainbow, then the edge $v_{i,j}v'_{i,j}$ must be colored with $2n$ colors. Thus, for the graph $M(F_{n,4})$ to be rainbow connected, it cannot be colored with $3n + 1$ colors, so the assumption is false and it is provem that $rc(M(F_{n,4})) = 3n + 2$. Next it will be shown that $rc(M(F_{n,4})) = 3n + 2$ with the definition of edge coloring c as follows:

$c(v_{i,j}v'_{i,j})$	$= i$	$i \in [1,n], j = 1$
$c(v_{i,j}v'_{i,j})$	$i + n$	$i \in [1,n], j = 2$
$c(v_i v'_{i,j})$	$i + 1$	$i \in [1,n], j \in [1,2]$
$c(v'_i v'_{i,j})$	$i + n$	$i \in [1,n], j = 1$
$c(v'_i v'_{i,j})$	i	$i \in [1,n], j = 2$
$c(v_i v'_i)$	$i + 2n$	$i \in [1,n]$
$c(u_i u'_i)$	1	$i \in [1,n-1]$
$c(u_{i+1}u'_i)$	n	$i \in [1,n-1]$
$c(u'_i u'_{i+1})$	$i + 1$	$i \in [1,n-2], n \geq 3$
$c(u_i v'_i)$	$v'_{i,2}) = i + 1$	$i \in [1,n]$
$c(u'_i v'_i)$	$3n + 1$	$i \in [1,n-1]$
$c(u'_i v'_{i+1})$	$3n + 2$	$i \in [1,n-1]$

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The rainbow coloring of the graph $M(F_{n,4})$ for $n \geq 2$ can be seen in figure 2 :

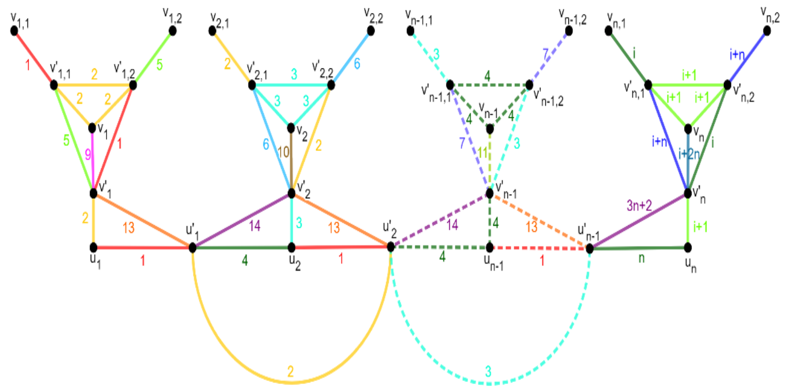

Figure 2. Rainbow Coloring $M(F_{n,4})$

Furthermore, the rainbow path for each pair $x, y \in E(G)$ with coloring c in the middle graph of the firecracker graph $M\left(F_{n,4}\right)$ is shown in table 1 :

Table 1. Rainbow Path $M(F_{n,4})$

No	x	y	Kondisi	Lintasan
1.	u_i	u_j	$i = [1, n-1], j = [i+1, n]$	$u_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, u_j$
2.	u'_i	u'_j	$i = [1, n-3], j = [i+2, n-1]$	$u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_j$
3.	u_i	u'_j	$i = [1, n-2], j = [i+1, n-1]$	$u_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_j$
	u'_i	u_j	$i = [1, n-2], j = [i+2, n]$	$u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, u_j$
4.	u_i	v_j	$i = [1, n-1], j = [i+1, n]$	$u_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j, v_j$
	v_i	u_j	$i = [1, n], j = [i, n]$	$v_i, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, u_j$
5.	u'_i	v'_j	$i = [1, n-2], j = [i+2, n]$	$u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j$
	v'_i	u'_j	$i = [1, n-2], j = [i+1, n-1]$	$v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_j$
6.	u_i	v'_j	$i = [1, n-1], j = [i+1, n]$	$u_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j$
	v'_i	u_j	$i = [1, n-1], j = [i+1, n]$	$v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, u_j$
7.	u'_i	v_j	$i = [1, n-1], j = [i+1, n]$	$u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j, v_j$
	v_i	u'_j	$i = [1, n-1], j = [i, n-1]$	$v_i, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_j$
8.	u_i	$v_{j,k}$	$i = [1, n-1], j = [i+1, n],$ $k = [1,2]$	$u_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j,$ $v'_{j,k}, v_{j,k}$
	$v_{i,k}$	u_j	$i = [1, n], j = [i, n], k = [1,2]$	$v_{i,k}, v'_{i,k}, v_i, u'_i, u'_{i+1}, u'_{i+2}, \dots,$ u'_{j-1}, u_j
9.	u'_i	$v'_{j,k}$	$i = [1, n-1], j = [i+1, n],$ $k = [1,2]$	$u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j, v'_{j,k}$
	$v'_{i,k}$	u'_j	$i = [1, n-1], j = [i, n-1],$ $k = [1,2]$	$v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_j$
10.	u_i	$v'_{j,k}$	$i = [1, n-1], j = [i+1, n],$ $k = [1,2]$	$u_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j,$ $v'_{j,k}$
	$v'_{i,k}$	u_j	$i = [1, n-1], j = [i+1, n],$ $k = [1,2]$	$v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1},$ u_j

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11.	u'_i	$v_{j,k}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j, v'_{j,k}$, $v_{j,k}$
	$v_{i,k}$	u'_j	$i=[1,n-1], j=[i,n-1]$, $k=[1,2]$	$v_{i,k}, v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots$, u'_j
12.	v_i	v_j	$i=[1,n-1], j=[i+1,n]$	$v_i, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, v'_j, v_j
13.	v'_i	v'_j	$i=[1,n-1], j=[i+1,n]$	$v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j$
14.	v_i	v'_j	$i=[1,n-1], j=[i+1,n]$	$v_i, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j$
	v'_i	v_j	$i=[1,n-1], j=[i+1,n]$	$v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j, v_j$
15.	v_i	$v_{j,k}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$v_i, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, $v'_j, v'_{j,k}, v_{j,k}$
	$v_{i,k}$	v_j	$i=[1,n-1], j=[i,n]$, $k=[1,2]$	$v_{i,k}, v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots$, u'_{j-1}, v'_j, v_j
16.	v'_i	$v'_{j,k}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, $v'_j, v'_{j,k}$
	$v'_{i,k}$	v'_j	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots$, u'_{j-1}, v'_j
17.	v_i	$v'_{j,k}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$v_i, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, $v'_j, v'_{j,k}$
	$v'_{i.k}$	v_j	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, v'_j, v_j
18.	v'_i	$v_{j,k}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$	$v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}, v'_j$, $v'_{j,k}, v_{j,k}$
	$v_{i,k}$	v'_j	$i=[1,n-1], j=[i,n]$, $k=[1,2]$	$v_{i,k}, v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots$, u'_{j-1}, v'_j
19.	$v'_{i,k}$	$v'_{j,l}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2], l=[1,2]$	$v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, $v'_j, v'_{j,l}$
20.	$v_{i,j}$	$v_{i,j+1}$	$i=[1,n], j=1$	$v_{i,j}, v'_{i,j}, v'_{i,j+1}, v_{i,j+1}$
21.	$v_{i,j}$	$v'_{i,j+1}$	$i=[1,n], j=1$	$v_{i,j}, v'_{i,j}, v'_{i,j+1}$
22.	$v'_{i,j}$	$v_{i,j+1}$	$i=[1,n], j=1$	$v'_{i,j}, v'_{i,j+1}, v_{i,j+1}$
23.	$v_{i,k}$	$v'_{j,l}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$, $l=[1,2]$	$v_{i,k}, v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots$, $u'_{j-1}, v'_j, v'_{j,l}$
	$v'_{i,k}$	$v_{j,l}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$, $l=[1,2]$	$v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots, u'_{j-1}$, $v'_j, v'_{j,l}, v_{j,l}$
24.	$v_{i,k}$	$v_{j,l}$	$i=[1,n-1], j=[i+1,n]$, $k=[1,2]$, $l=[1,2]$	$v_{i,k}, v'_{i,k}, v'_i, u'_i, u'_{i+1}, u'_{i+2}, \dots$, $u'_{j-1}, v'_j, v'_{j,l}, v_{j,l}$

Based on the definition of rainbow coloring and rainbow trajectory in table 1 then theorem 2 which states $rc(M(F_{n,4})) = 3n + 2$ with $n \geq 2$ is proved.

CONCLUSION

Based on the results and discussion, it can be concluded that the rainbow connected number of the middle graph $M(G)$ of the firecrackers graph $(F_{n,4})$ with $n \geq 2$, where n is an integer, then $rc(M(F_{n,4})) = 3n + 2$.

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