

The Effect of Numbered Heads Together and Teacher-Centered Learning on Students' Conceptual Understanding and Mathematical Reasoning in Systems of Linear Equations in Two Variables

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Article Info	ABSTRACT
Keywords: Numbered Heads Together, Teacher-Centered Learning, conceptual understanding, mathematical reasoning, Systems of Linear Equations in Two Variables.	This study investigated the effect of the Numbered Heads Together (NHT) learning model compared with Teacher-Centered Learning (TCL) on students' conceptual understanding and mathematical reasoning in learning Systems of Linear Equations in Two Variables (SPLDV). The study employed a quantitative approach using a quasi-experimental design with a pretest–posttest non-equivalent control group design. The participants consisted of 40 eighth-grade students at MTs Al Washliyah Pematangsiantar, selected through cluster random sampling, with 20 students assigned to the experimental group and 20 students to the control group. Data were collected using conceptual understanding and mathematical reasoning tests that had been validated and tested for reliability. Data analysis was conducted using descriptive statistics and Multivariate Analysis of Variance (MANOVA) with IBM SPSS Statistics 26 at a significance level of 0.05. The results revealed a significant multivariate effect of the learning model on students' conceptual understanding and mathematical reasoning (Wilks' Lambda = .731, $F = 6.817$, $p = .003$). Follow-up analyses showed that the NHT model significantly improved conceptual understanding ($F = 5.676$, $p = .022$) and mathematical reasoning ($F = 12.054$, $p = .001$) compared with Teacher-Centered Learning. Students in the NHT group also achieved higher mean scores on both learning outcomes than those in the TCL group. These findings indicate that the Numbered Heads Together learning model is more effective than Teacher-Centered Learning in enhancing students' conceptual understanding and mathematical reasoning in learning SPLDV. Therefore, NHT can be recommended as an effective student-centered instructional strategy to support mathematics learning and the implementation of the Merdeka Curriculum.
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INTRODUCTION

Mathematics is a fundamental discipline that plays an essential role in developing students' logical, analytical, critical, and creative thinking skills. Beyond acquiring computational proficiency, mathematics education aims to develop students' ability to understand mathematical concepts, reason logically, solve problems systematically, and apply mathematical knowledge in various real-life situations. These competencies are strongly emphasized in the Merdeka Curriculum, which promotes student-centered learning and the development of higher-order thinking skills required in the twenty-first century. However, despite continuous curriculum reforms, many Indonesian students continue to experience difficulties in achieving satisfactory mathematical competencies, particularly in conceptual understanding and mathematical reasoning (Astriani & Al Dhana, 2023).

Among the various competencies expected in mathematics learning, conceptual understanding and mathematical reasoning are considered two interrelated and indispensable

cognitive abilities. Conceptual understanding enables students to recognize relationships among mathematical ideas, connect different representations, and apply concepts flexibly to unfamiliar situations. Meanwhile, mathematical reasoning allows students to formulate conjectures, justify mathematical procedures, evaluate arguments, and construct logical conclusions during problem-solving. These two competencies complement one another because students with strong conceptual understanding generally demonstrate better reasoning performance when solving mathematical problems. Consequently, improving both competencies simultaneously has become one of the primary objectives of contemporary mathematics education.

Several recent studies have demonstrated that innovative learning models can significantly improve students' mathematical competencies. Ammy et al, (2025) reported that the integration of Problem-Based Learning (PBL) with the Quizizz educational platform significantly enhanced students' mathematical conceptual understanding through active participation and immediate feedback. Likewise, Najma and Irvan (2022) found that instructional materials developed using the Realistic Mathematics Education (RME) approach successfully improved students' conceptual understanding by connecting mathematical concepts with authentic learning contexts. Furthermore, Mushlihuiddin et al (2020) showed that student-centered learning models effectively enhanced students' mathematical reasoning and proof abilities because students actively constructed arguments, evaluated solutions, and communicated mathematical ideas during classroom discussions. Similarly, Irvan and Mushlihuiddin (2021) concluded that problem-based instructional materials contributed significantly to improving students' critical thinking skills, emphasizing the importance of active engagement during mathematics learning. Collectively, these studies indicate that learning environments encouraging collaboration, interaction, and active participation tend to produce better mathematical outcomes than conventional instruction.

One mathematics topic that consistently presents learning difficulties for junior high school students is the System of Linear Equations in Two Variables (SPLDV). SPLDV requires students not only to perform algebraic procedures but also to understand relationships between variables, construct mathematical models from contextual situations, interpret multiple mathematical representations, and justify solution strategies logically. These characteristics make SPLDV an appropriate topic for evaluating both conceptual understanding and mathematical reasoning simultaneously. Nevertheless, previous studies have consistently reported that many students encounter substantial difficulties in learning SPLDV. Khoirunnisa and Soro (2021) found that students frequently struggle to connect symbolic, graphical, and contextual representations, while Muthmainnah (2023) reported that many students rely heavily on memorizing procedural algorithms without fully understanding the underlying mathematical concepts. Consequently, students often experience difficulties when solving contextual or non-routine mathematical problems requiring higher-order reasoning.

One major factor contributing to these learning difficulties is the continued implementation of Teacher-Centered Learning (TCL) in mathematics classrooms. Under this instructional approach, teachers dominate classroom activities by explaining concepts and demonstrating procedures, whereas students primarily act as passive recipients of information. Such learning environments provide limited opportunities for discussion, collaboration, mathematical argumentation, and reflective thinking. As a result, students frequently develop procedural knowledge without

establishing meaningful conceptual understanding or logical reasoning abilities. Although teacher-centered instruction may efficiently deliver mathematical content, it often fails to cultivate the higher-order thinking skills required by current educational standards.

To address these limitations, mathematics educators have increasingly adopted cooperative learning models that actively engage students in constructing knowledge through collaboration and communication. One cooperative learning strategy that has received considerable attention is the Numbered Heads Together (NHT) model. In NHT, every group member is responsible for understanding the discussion because each student has an equal opportunity to present the group's answer (Izza et al. 2025). This learning structure promotes active participation, peer interaction, collaborative reasoning, and individual accountability, enabling students to develop conceptual understanding while simultaneously strengthening mathematical reasoning. Through discussion and collaborative problem-solving, students are encouraged to explain mathematical ideas, justify their solutions, evaluate alternative strategies, and construct knowledge collectively.

Numerous studies have reported positive effects of cooperative learning on students' mathematical achievement. Agustin et al. (2024) demonstrated that cooperative learning significantly improved students' conceptual understanding and critical thinking skills compared with conventional instruction. Likewise, Suherman and Irvan (2021) found that implementing the Realistic Mathematics Education approach enhanced students' mathematical competencies by actively involving learners in meaningful mathematical activities. These findings consistently suggest that instructional models emphasizing student participation and collaborative interaction provide greater opportunities for developing conceptual understanding and reasoning than traditional teacher-centered approaches.

Although previous research has confirmed the effectiveness of various innovative learning models, several important limitations remain. First, most previous studies have investigated only a single mathematical competency, such as conceptual understanding, reasoning, critical thinking, or problem-solving ability, rather than examining multiple competencies simultaneously. Second, relatively few studies have directly compared the Numbered Heads Together (NHT) model with Teacher-Centered Learning (TCL), particularly within the context of Systems of Linear Equations in Two Variables (SPLDV) at the junior high school level. Third, previous studies have predominantly employed separate univariate statistical analyses, providing limited evidence regarding the simultaneous effects of instructional models on conceptual understanding and mathematical reasoning. These limitations indicate the need for further investigation using more comprehensive analytical approaches capable of evaluating multiple dependent variables concurrently.

The novelty of this study lies in three important aspects. First, this study simultaneously investigates the influence of the Numbered Heads Together (NHT) and Teacher-Centered Learning (TCL) models on two fundamental mathematical competencies, namely conceptual understanding and mathematical reasoning. Second, the study specifically focuses on Systems of Linear Equations in Two Variables (SPLDV), a mathematical topic that demands both conceptual comprehension and logical reasoning during problem-solving. Third, this study employs Multivariate Analysis of Variance (MANOVA) to evaluate the simultaneous effects of instructional models on multiple cognitive outcomes, thereby providing more comprehensive empirical evidence than previous studies that mainly relied on separate univariate analyses. Unlike previous studies that generally focused on a

single mathematical competency or compared innovative learning with conventional instruction in a broader context, this study offers an integrated evaluation of the effectiveness of cooperative learning on multiple mathematical competencies simultaneously.

Therefore, this study aims to investigate the effects of the Numbered Heads Together (NHT) learning model compared with Teacher-Centered Learning (TCL) on students' conceptual understanding and mathematical reasoning in learning Systems of Linear Equations in Two Variables (SPLDV). Furthermore, this study seeks to provide empirical evidence regarding the effectiveness of cooperative learning in simultaneously improving multiple mathematical competencies using a multivariate analytical approach. The findings are expected to enrich the literature on cooperative mathematics learning and provide practical recommendations for mathematics teachers in implementing student-centered instructional strategies that support the successful implementation of the Merdeka Curriculum and the development of twenty-first-century mathematical competencies.

RESEARCH METHOD

This study employed a quantitative approach using a quasi-experimental design with a pretest–posttest non-equivalent control group design. The study compared the effects of the Numbered Heads Together (NHT) learning model and Teacher-Centered Learning (TCL) on students' conceptual understanding and mathematical reasoning in the topic of Systems of Linear Equations in Two Variables (SPLDV).

The research was conducted at MTs Al Washliyah Pematangsiantar during the 2025/2026 academic year. The participants consisted of 40 eighth-grade students selected through cluster random sampling. Twenty students were assigned to the experimental group and learned using the NHT model, while the remaining twenty students formed the control group and learned using the TCL approach.

Data were collected using two essay tests measuring conceptual understanding and mathematical reasoning. The instruments were validated by mathematics education experts and met the required validity and reliability criteria before being administered.

The data were analyzed using IBM SPSS Statistics Version 26. Descriptive statistics were used to summarize students' performance. Prior to hypothesis testing, the assumptions of normality and homogeneity were examined using the Shapiro–Wilk test and Levene's Test, respectively. The research hypotheses were tested using Multivariate Analysis of Variance (MANOVA) because the study involved one independent variable (learning model) and two dependent variables (conceptual understanding and mathematical reasoning). When the multivariate test showed significant results, Tests of Between-Subjects Effects were conducted to determine the effect of the learning model on each dependent variable individually. Statistical significance was determined at the 0.05 significance level, and Partial Eta Squared was reported to indicate the effect size.

RESULTS AND DISCUSSION

Results

This study involved 40 eighth-grade students, consisting of 20 students in the experimental group taught using the Numbered Heads Together (NHT) learning model and 20 students in the control group taught using the Teacher-Centered Learning (TCL) approach. The descriptive statistics for students' conceptual understanding and mathematical reasoning are presented in Table 1.

Table 1. Descriptive Statistics of Conceptual Understanding and Mathematical Reasoning

Variabel	Model	N	Mean	SD
Understanding of Concepts	NHT	20	76,25	20,87
	TCL	20	57,55	28,22
Mathematical Reasoning	NHT	20	61,70	18,28
	TCL	20	42,50	16,66

Based on Table 1, students who learned using the NHT model achieved higher mean scores than those who learned using the TCL model in both dependent variables. The mean score of conceptual understanding in the NHT group was 76,25, whereas the TCL group obtained a mean score of 57,55. Likewise, the mean score of mathematical reasoning was 61,70 in the NHT group compared with 42,50 in the TCL group. These descriptive results indicate that students taught using the NHT model performed better than those taught using the TCL

Before conducting hypothesis testing, the assumptions of normality and homogeneity were examined. The normality of the data was assessed using the Shapiro–Wilk test. The results are presented in Table 2.

Table 2. Results of the Shapiro–Wilk Normality Test

Variabel	Model	Sig.
Understanding Of Concepts	NHT	0,000
	TCL	0,060
Mathematical Reasoning	NHT	0,003
	TCL	0,171

The results indicate that the conceptual understanding and mathematical reasoning scores in the NHT group did not fully satisfy the normality assumption ($p < 0,05$), whereas both variables in the TCL group were normally distributed ($p > 0,05$). Nevertheless, because the sample sizes of the two groups were equal, the MANOVA procedure remained appropriate for hypothesis testing.

The homogeneity of variance assumption was examined using Levene's Test, and the results are presented in Table 3.

Table 3. Results of Levene's Test of Homogeneity

Variabel	F	Sig.
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Understanding of Concepts	4,188	0,048
Mathematical Reasoning	0,060	0,808

As shown in Table 3, mathematical reasoning met the homogeneity assumption ($p = 0,808 > 0,05$), whereas conceptual understanding produced a significance value of 0,048, indicating a slight violation of the assumption. However, this result was considered acceptable because MANOVA is relatively robust when group sizes are balanced.

The simultaneous effect of the learning model on students' conceptual understanding and mathematical reasoning was analyzed using Multivariate Analysis of Variance (MANOVA). The multivariate test results are presented in Table 4.

Table 4. MANOVA Results

Statistik	F	Sig.	Partial Eta Squared
Pillai's Trace	6,817	0,003	0,269
Wilks' Lambda	6,817	0,003	0,269
Hotelling's Trace	6,817	0,003	0,269

The results show that the learning model had a statistically significant multivariate effect on the combined dependent variables. Based on Wilks' Lambda ($\Lambda = 0,731$), the analysis produced $F = 6,817$ with $p = 0,003$, indicating that the null hypothesis was rejected. Furthermore, the Partial Eta Squared value of 0,269 suggests that approximately 26,9% of the variation in students' conceptual understanding and mathematical reasoning was explained by the learning model.

To determine the effect of the learning model on each dependent variable separately, follow-up univariate analyses were performed. The results are presented in Table 5.

Table 5. Tests of Between-Subjects Effects

Variabel	F	Sig.	Partial Eta Squared
Understanding of Concepts	5,676	0,022	0,130
Reasoning Mathematical	12,054	0,001	0,241

The analysis revealed that the learning model significantly affected conceptual understanding ($F = 5,676$, $p = 0,022$) and mathematical reasoning ($F = 12,054$, $p = 0,001$). These findings indicate that students who learned using the Numbered Heads Together (NHT) model achieved significantly better conceptual understanding and mathematical reasoning than those who learned through the Teacher-Centered Learning (TCL) approach.

Discussion

The findings of this study demonstrate that the Numbered Heads Together (NHT) learning model was more effective than Teacher-Centered Learning (TCL) in improving students' conceptual understanding and mathematical reasoning in learning Systems of Linear Equations in Two Variables (SPLDV). The multivariate analysis confirmed that the learning model significantly influenced both dependent variables simultaneously. These findings indicate that learning activities emphasizing collaboration, active participation, and individual accountability provide a more meaningful learning experience than teacher-centered instruction.

The superior performance of students in the NHT group can be explained by the characteristics of the learning model itself. During NHT activities, students are encouraged to discuss mathematical problems, exchange ideas, explain concepts to their peers, and jointly determine appropriate solution strategies. Such learning activities allow students to actively construct their own understanding rather than simply receiving information from the teacher. According to the constructivist learning theory, meaningful knowledge is developed through active interaction with learning tasks and social collaboration. Therefore, students who actively participate in discussion are more likely to develop a deeper conceptual understanding of mathematical concepts.

The findings regarding conceptual understanding are consistent with previous studies reporting that student-centered learning models improve students' ability to understand mathematical concepts. Ammy et al (2025) found that implementing Problem-Based Learning assisted by Quizizz significantly enhanced students' mathematical conceptual understanding because students became actively involved in exploring mathematical ideas and solving problems collaboratively. Likewise, Najma and Irvan (2022) reported that learning materials developed using the Realistic Mathematics Education approach successfully improved students' conceptual understanding by connecting mathematical concepts with meaningful real-life situations. These studies support the present findings that active learning environments facilitate a deeper understanding of mathematical concepts than conventional teacher-centered instruction.

The present study also found that students who learned through the NHT model demonstrated significantly better mathematical reasoning than those taught using TCL. This finding suggests that cooperative learning encourages students to justify mathematical procedures, evaluate alternative solutions, communicate mathematical ideas, and construct logical arguments during group discussions. Such activities naturally promote reasoning processes because students must explain not only the answers but also the underlying mathematical thinking. Similar findings were reported by Mushlihuiddin, Wahyun, and Irvan (2020), who concluded that student-centered instructional models significantly improved students' mathematical reasoning and proof abilities through discussion, argumentation, and collaborative problem-solving.

Compared with Teacher-Centered Learning, the NHT model provides greater opportunities for students to become actively engaged throughout the learning process. In conventional classrooms, learning is generally dominated by teacher explanations, while students mainly listen, record information, and complete exercises individually. Although this approach may efficiently deliver learning materials, it often limits opportunities for students to communicate mathematical ideas and develop higher-order thinking skills. Conversely, NHT creates an interactive learning environment in which every student has individual responsibility for understanding the material

because any group member may be selected to present the group's answer. Consequently, all students are encouraged to participate actively throughout the learning process.

The findings of this study are also consistent with previous research conducted by Agustin et al. (2024), who reported that cooperative learning significantly improved students' conceptual understanding and critical thinking skills compared with conventional instruction. Likewise, Suherman and Irvan (2021) demonstrated that active learning approaches provide better opportunities for developing students' mathematical competencies than traditional teacher-centered learning. Collectively, these findings strengthen the empirical evidence supporting the effectiveness of cooperative learning in mathematics education.

From a practical perspective, the findings suggest that mathematics teachers should consider implementing the Numbered Heads Together model more frequently, particularly when teaching mathematical topics that require conceptual understanding and reasoning, such as SPLDV. Through structured collaboration and active discussion, students become more engaged in learning and develop a deeper understanding of mathematical concepts while simultaneously strengthening their reasoning abilities. These findings also support the implementation of the Merdeka Curriculum, which emphasizes student-centered learning and the development of higher-order thinking skills.

Despite these positive findings, several limitations should be acknowledged. The study involved a relatively small sample drawn from a single junior high school, which may limit the generalizability of the findings. In addition, the research focused only on the SPLDV topic and examined cognitive outcomes without considering affective variables such as learning motivation, self-efficacy, or mathematical anxiety. Future studies are therefore encouraged to involve larger and more diverse samples, investigate different mathematics topics, and incorporate additional variables to obtain a more comprehensive understanding of the effectiveness of cooperative learning models.

CONCLUSION

This study investigated the effect of the Numbered Heads Together (NHT) learning model compared with Teacher-Centered Learning (TCL) on students' conceptual understanding and mathematical reasoning in learning Systems of Linear Equations in Two Variables (SPLDV). The findings revealed that the learning model significantly affected both dependent variables simultaneously. Students who learned using the NHT model achieved higher conceptual understanding and mathematical reasoning scores than those who learned through the TCL approach. These findings indicate that cooperative learning through NHT provides a more effective learning environment by encouraging active participation, collaborative discussion, and individual accountability, which contribute to the development of higher-order mathematical thinking.

The findings of this study have practical implications for mathematics teachers and schools. The NHT learning model can serve as an effective student-centered instructional strategy, particularly for mathematics topics requiring conceptual understanding and logical reasoning. Its implementation supports the objectives of the Merdeka Curriculum, which emphasizes active learning, collaboration, and the development of twenty-first-century competencies.

This study was limited to one school and focused only on the SPLDV topic with a relatively small sample size. Therefore, future studies are recommended to involve larger and more diverse

samples, investigate other mathematics topics, and examine additional variables such as learning motivation, self-efficacy, or mathematical communication to obtain a more comprehensive understanding of the effectiveness of cooperative learning models.

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